1]

Exam III Solutions

For an arbitrary direction (θ, ϕ) we get eigen-spinors using Eq. 4.155 as

$$\chi_+ = \begin{bmatrix} \cos(\theta/2) \\ e^{i\phi} \sin(\theta/2) \end{bmatrix}, \quad \chi_- = \begin{bmatrix} e^{-i\phi} \sin(\theta/2) \\ -\cos(\theta/2) \end{bmatrix}$$

For x axis, $\theta = \pi/2$, $\phi = 0$

$$\Rightarrow \chi_\pm^{(x)} = \begin{bmatrix} 1 \\ \pm 1 \end{bmatrix} \frac{1}{\sqrt{2}}$$

So first measurement gives $\hbar/2$ with unit probability. Spinor collapses to $\chi_+^{(x)}$.

For y axis, $\theta = \pi/2$, $\phi = \pi/2$

$$\Rightarrow \chi_+^{(y)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}, \quad \chi_-^{(y)} = \frac{-1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}$$

Probability of getting $\pm\hbar/2$ on second measurement is

$$|\chi_\pm^{(y)\dagger} \chi_+^{(x)}|^2 = \left|\frac{1 \pm i}{2}\right|^2 = 1/2$$

Now spinor collapses to $\chi_\pm^{(y)}$.

Case (i) Collapse to $\chi_+^{(y)}$

$\Rightarrow$ Probability of measuring $\hbar/2$ on third measurement is $|1/\sqrt{2}|^2 = 1/2$

Same logic for $-\hbar/2$. $P = \sqrt{|i/\sqrt{2}|^4} = \sqrt{1/4} = 1/2$

Case (ii) Collapse to $\chi_-^{(y)}$

$\Rightarrow$ $P_\pm = 1/2$ again.

Combine all cases.

$P(S_x = \hbar/2) = 1$, $P(S_x = -\hbar/2) = 0$

$P(S_y = \pm\hbar/2) = 1/2$

$P(S_z = \pm\hbar/2) = \frac{1}{4} + \frac{1}{4} = 1/2$.

2] Notation is $P \Rightarrow L=1$, $D \Rightarrow L=2$, $F \Rightarrow L=3$,

$^{2S+1}L_J$, where $J = |(L-S)|, |L-S|+1, \dots L+S$

(i) $^3P_0 \Rightarrow L=1, \; S=1, \; J=|L-S|=0$

(ii) $^5D_3 \Rightarrow L=2, \; S=2, \quad J=3= L+S-1 > |L-S|=0$

(iii) $^3F_1 \Rightarrow L=3, \; S=1, \; J=1 < |L-S|=2$

$\Rightarrow$ this case cannot be valid.

3 (a) Degeneracy is $2S+1=4$, $S=3/2$.

So 4 electrons can have same spatial wavefunctions.

(b) $\frac{q}{2S+1} = 8/(2S+1) = 2$, $\Rightarrow$ filled bands

$\Rightarrow$ insulator or semiconductor.

(c) $\frac{q}{2S+1} = \frac{13}{4} = 3.25$, $\Rightarrow$ unfilled bands

$\Rightarrow$ conductor.