4] (a) False , because $[\hat{L}_x, \hat{L}^2] = [\hat{L}_x, \hat{H}] = [\hat{L}^2, \hat{H}] = 0$

(b) True , because $[\hat{L}_y, \hat{L}^2] = 0$ , but $[\hat{L}_x, \hat{L}_y] \neq 0$

(c) False , " $[\hat{L}_x, \hat{L}_z] \neq 0$

(d) True , " $[\hat{L}_y, \hat{H}] = 0$

(e) True , " ~~D~~ same logic as (a).

3 (a)

$$H_i = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2 x^2}{2}$$

(b)

$$\bar{E} = \mathcal{E}\hat{i} = -\hat{i}\frac{\partial V(x)}{\partial x} \Rightarrow V(x) = -\mathcal{E}x$$

$$\Rightarrow H_f = H_i - \mathcal{E}xq = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m\omega^2 x^2}{2} - \mathcal{E}xq$$

(c) $E_n^{(i)} = \hbar\omega\left[n + \frac{1}{2}\right]$, $n = 0, 1, 2, \cdots$

(d)

$$V_f(x) = \frac{m\omega^2 x^2}{2} - \mathcal{E}xq = \frac{m\omega^2 x^2}{2} - \mathcal{E}'x + \frac{\mathcal{E}'^2}{2m\omega^2} - \frac{\mathcal{E}'^2}{2m\omega^2}$$

$$= \left[\left(\frac{m\omega^2}{2}\right)^{1/2} x - \frac{\mathcal{E}'}{(2m\omega^2)^{1/2}}\right]^2 - \frac{\mathcal{E}'^2}{2m\omega^2}, \quad \mathcal{E}' \equiv q\mathcal{E}$$

$$= \frac{m\omega^2}{2}\left[x - \frac{\mathcal{E}'}{m\omega^2}\right]^2 - \frac{\mathcal{E}'^2}{2m\omega^2}$$

Let $y \equiv x - \frac{\mathcal{E}}{m\omega^2} \Rightarrow \frac{d^2}{dx^2} \equiv \frac{d^2}{dy^2}$

$$\Rightarrow H_f = -\frac{\hbar^2}{2m}\frac{d^2}{dy^2} + \frac{m\omega^2 y^2}{2} - \frac{\mathcal{E}'^2}{2m\omega^2}$$

$$\Rightarrow E_n^{(f)} = \hbar\omega\left[n + \frac{1}{2}\right] - \frac{\mathcal{E}^2 q^2}{2m\omega^2}$$

Interpretation: $\mathcal{E}$ just shifts the zero position or equilibrium position of the oscillator. It also reduces the energy by $\frac{\mathcal{E}^2 q^2}{2m\omega^2}$.

5 $$\frac{dE(\omega)}{V} = \frac{\hbar\omega^3 d\omega}{\pi^2 c^3 \left[e^{\frac{\hbar\omega}{k_B T}} - 1\right]}$$

5, a $$\Rightarrow E/V = \int_0^\infty \frac{dE(\omega)}{V} = \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3 d\omega}{\left[e^{\frac{\hbar\omega}{k_B T}} - 1\right]}$$

$$= \frac{\hbar}{\pi^2 c^3} \times \left(\frac{k_B T}{\hbar}\right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

$$= k_B^4 T^4 \hbar^{-3} c^{-3} G \text{ where}$$

$$G = \pi^{-2} \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

5, b $\alpha = \beta = -3$, $\gamma = \delta = 4$.

4]

$$\Psi(x,0) = Nx \,, \quad 0 < x \le L/2$$
$$= N(L-x)\,, \quad L/2 < x \le L$$

(a) $\int |\Psi|^2 dx = 1 \Rightarrow N^2\left[\int_0^{L/2} x^2 dx + \int_{L/2}^{L} (L-x)^2 dx\right] = 1$

$$\Rightarrow N^2\left[2\int_0^{L/2} x^2 dx\right] = 1 \Rightarrow \frac{2N^2}{8}\frac{L^3}{3} = 1 \Rightarrow \frac{N}{\sqrt{8}} = \left[\frac{3}{2L^3}\right]^{1/2}$$

$$\Rightarrow N = \left[\frac{12}{L^3}\right]^{1/2} = 2\left[\frac{3}{L^3}\right]^{1/2}$$

(b) The energy wavefunctions are $\psi_n(x) = \left(\frac{2}{L}\right)^{1/2} \sin\left(\frac{n\pi x}{L}\right)$

$n = 1, 2, \ldots$ and $0 < x < L$

$= 0$, otherwise.

$$\Psi(x,0) = \sum_{n=1}^{\infty} c_n \psi_n(x)$$

$$\Rightarrow c_n = \int_0^L \psi_n^*(x)\, \Psi(x,0)\, dx$$

$$\Rightarrow c_1 = \frac{2\sqrt{6}}{L^2} I_1 \,, \quad I_1 \equiv I_{11} + I_{12}$$

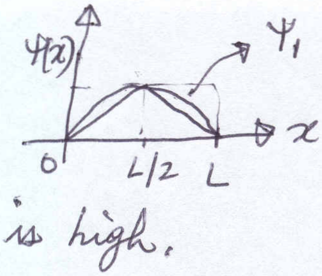
$$I_{11} \equiv \int_0^{L/2} x \sin\left(\frac{\pi x}{L}\right) dx = \frac{L^2}{\pi^2}\int_0^{\pi/2} y \sin y \, dy = \frac{L^2}{\pi^2}\left\{\int_0^{\pi/2} \cos y \, dy\right\} = \frac{L^2}{\pi^2}$$

$$I_{12} = \int_{L/2}^{L} (L-x) \sin\left(\frac{\pi x}{L}\right) dx = \int_0^{L/2} y \sin\left(\pi - \frac{\pi y}{L}\right) dy = \int_0^{L/2} y \sin\left(\frac{\pi y}{L}\right) dy = \frac{L^2}{\pi^2}$$

$$\Rightarrow c_1 = \frac{2\sqrt{6}}{L^2} \times \frac{2L^2}{\pi^2} = \frac{4\sqrt{6}}{\pi^2}$$

$$P(E = E_1) = |c_1|^2 = \frac{96}{\pi^4} \,, \quad E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

$\approx 1 \longrightarrow$ because




overlap is high.