

Problem 1.1

(a)

$$\langle j \rangle^2 = 21^2 = \boxed{441.}$$

$$\begin{aligned}\langle j^2 \rangle &= \frac{1}{N} \sum j^2 N(j) = \frac{1}{14} [(14^2) + (15^2) + 3(16^2) + 2(22^2) + 2(24^2) + 5(25^2)] \\ &= \frac{1}{14} (196 + 225 + 768 + 968 + 1152 + 3125) = \frac{6434}{14} = \boxed{459.571.}\end{aligned}$$

(b)

j	$\Delta j = j - \langle j \rangle$
14	$14 - 21 = -7$
15	$15 - 21 = -6$
16	$16 - 21 = -5$
22	$22 - 21 = 1$
24	$24 - 21 = 3$
25	$25 - 21 = 4$

$$\begin{aligned}\sigma^2 &= \frac{1}{N} \sum (\Delta j)^2 N(j) = \frac{1}{14} [(-7)^2 + (-6)^2 + (-5)^2 \cdot 3 + (1)^2 \cdot 2 + (3)^2 \cdot 2 + (4)^2 \cdot 5] \\ &= \frac{1}{14} (49 + 36 + 75 + 2 + 18 + 80) = \frac{260}{14} = \boxed{18.571.}\end{aligned}$$

$$\sigma = \sqrt{18.571} = \boxed{4.309.}$$

(c)

$$\langle j^2 \rangle - \langle j \rangle^2 = 459.571 - 441 = 18.571. \quad [\text{Agrees with (b).}]$$

Problem 1.3

(a)

$$1 = \int_{-\infty}^{\infty} A e^{-\lambda(x-a)^2} dx. \quad \text{Let } u \equiv x - a, \quad du = dx, \quad u: -\infty \rightarrow \infty.$$

$$1 = A \int_{-\infty}^{\infty} e^{-\lambda u^2} du = A \sqrt{\frac{\pi}{\lambda}} \Rightarrow \boxed{A = \sqrt{\frac{\lambda}{\pi}}}.$$

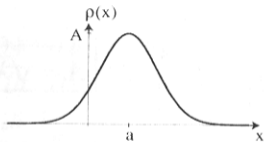
(b)

$$\begin{aligned} \langle x \rangle &= A \int_{-\infty}^{\infty} x e^{-\lambda(x-a)^2} dx = A \int_{-\infty}^{\infty} (u+a) e^{-\lambda u^2} du \\ &= A \left[\int_{-\infty}^{\infty} u e^{-\lambda u^2} du + a \int_{-\infty}^{\infty} e^{-\lambda u^2} du \right] = A \left(0 + a \sqrt{\frac{\pi}{\lambda}} \right) = \boxed{a}. \end{aligned}$$

$$\begin{aligned} \langle x^2 \rangle &= A \int_{-\infty}^{\infty} x^2 e^{-\lambda(x-a)^2} dx \\ &= A \left\{ \int_{-\infty}^{\infty} u^2 e^{-\lambda u^2} du + 2a \int_{-\infty}^{\infty} u e^{-\lambda u^2} du + a^2 \int_{-\infty}^{\infty} e^{-\lambda u^2} du \right\} \\ &= A \left[\frac{1}{2\lambda} \sqrt{\frac{\pi}{\lambda}} + 0 + a^2 \sqrt{\frac{\pi}{\lambda}} \right] = \boxed{a^2 + \frac{1}{2\lambda}}. \end{aligned}$$

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = a^2 + \frac{1}{2\lambda} - a^2 = \frac{1}{2\lambda}; \quad \boxed{\sigma = \frac{1}{\sqrt{2\lambda}}}.$$

(c)



Problem 1.5

(a)

$$1 = \int |\Psi|^2 dx = 2|A|^2 \int_0^\infty e^{-2\lambda x} dx = 2|A|^2 \left(\frac{e^{-2\lambda x}}{-2\lambda} \right) \Big|_0^\infty = \frac{|A|^2}{\lambda}; \quad \boxed{A = \sqrt{\lambda}}$$

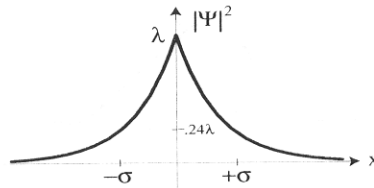
(b)

$$\langle x \rangle = \int x |\Psi|^2 dx = |A|^2 \int_{-\infty}^\infty x e^{-2\lambda|x|} dx = \boxed{0} \quad [\text{Odd integrand.}]$$

$$\langle x^2 \rangle = 2|A|^2 \int_0^\infty x^2 e^{-2\lambda x} dx = 2\lambda \left[\frac{2}{(2\lambda)^3} \right] = \boxed{\frac{1}{2\lambda^2}}$$

(c)

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{2\lambda^2}; \quad \boxed{\sigma = \frac{1}{\sqrt{2\lambda}}} \quad |\Psi(\pm\sigma)|^2 = |A|^2 e^{-2\lambda\sigma} = \lambda e^{-2\lambda/\sqrt{2\lambda}} = \lambda e^{-\sqrt{2}} = 0.2431$$



Probability outside:

$$2 \int_\sigma^\infty |\Psi|^2 dx = 2|A|^2 \int_\sigma^\infty e^{-2\lambda x} dx = 2\lambda \left(\frac{e^{-2\lambda x}}{-2\lambda} \right) \Big|_\sigma^\infty = e^{-2\lambda\sigma} = \boxed{e^{-\sqrt{2}} = 0.2431}$$