Problem 5.4

(a)

$$1 = \int |\psi_\pm|^2 d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= |A|^2 \int [\psi_a(\mathrm{r}_1)\psi_b(\mathrm{r}_2) \pm \psi_b(\mathrm{r}_1)\psi_a(\mathrm{r}_2)]^* [\psi_a(\mathrm{r}_1)\psi_b(\mathrm{r}_2) \pm \psi_b(\mathrm{r}_1)\psi_a(\mathrm{r}_2)]\, d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= |A|^2 \left[\int |\psi_a(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_b(\mathrm{r}_2)|^2 d^3\mathrm{r}_2 \pm \int \psi_a(\mathrm{r}_1)^*\psi_b(\mathrm{r}_1) d^3\mathrm{r}_1 \int \psi_b(\mathrm{r}_2)^*\psi_a(\mathrm{r}_2) d^3\mathrm{r}_2\right.$$

$$\left.\pm \int \psi_b(\mathrm{r}_1)^*\psi_a(\mathrm{r}_1) d^3\mathrm{r}_1 \int \psi_a(\mathrm{r}_2)^*\psi_b(\mathrm{r}_2) d^3\mathrm{r}_2 + \int |\psi_b(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_a(\mathrm{r}_2)|^2 d^3\mathrm{r}_2\right]$$

$$= |A|^2(1\cdot 1 \pm 0\cdot 0 \pm 0\cdot 0 + 1\cdot 1) = 2|A|^2 \Longrightarrow \boxed{A = 1/\sqrt{2}.}$$

(b)

$$1 = |A|^2 \int [2\psi_a(\mathrm{r}_1)\psi_a(\mathrm{r}_2)]^* [2\psi_a(\mathrm{r}_1)\psi_a(\mathrm{r}_2)]\, d^3\mathrm{r}_1 d^3\mathrm{r}_2$$

$$= 4|A|^2 \int |\psi_a(\mathrm{r}_1)|^2 d^3\mathrm{r}_1 \int |\psi_a(\mathrm{r}_2)|^2 d^3\mathrm{r}_2 = 4|A|^2. \qquad \boxed{A = 1/2.}$$

Problem 5.17

$$P = \frac{(3\pi^2)^{2/3}\hbar^2}{5m}\left(\frac{Nq}{V}\right)^{5/3} = AV^{-5/3} \Rightarrow B = -V\frac{dP}{dV} = -VA\left(\frac{-5}{3}\right)V^{-5/3-1} = \frac{5}{3}AV^{-5/3} = \frac{5}{3}P.$$

For copper, $B = \frac{5}{3}(3.84 \times 10^{10}\,\text{N/m}^2) = \boxed{6.4 \times 10^{10}\ \text{N/m}^2.}$

Problem 5.18

(a) Equations 5.59 and 5.63 $\Rightarrow \psi = A\sin kx + B\cos kx;\quad A\sin ka = \left[e^{iKa} - \cos ka\right]B.$ So

$$\psi = A\sin kx + \frac{A\sin ka}{(e^{iKa} - \cos ka)}\cos kx = \frac{A}{(e^{iKa} - \cos ka)}\left[e^{iKa}\sin kx - \sin kx\cos ka + \cos kx\sin ka\right]$$

$$= C\left\{\sin kx + e^{-iKa}\sin[k(a-x)]\right\},\ \text{where } C \equiv \frac{Ae^{iKa}}{e^{iKa} - \cos ka}.$$

(b) If $z = ka = j\pi$, then $\sin ka = 0$, Eq. 5.64 $\Rightarrow \cos Ka = \cos ka = (-1)^j \Rightarrow \sin Ka = 0$, so $e^{iKa} = \cos Ka + i\sin Ka = (-1)^j$, and the constant C involves division by zero. In this case we must go back to Eq. 5.63, which is a tautology (0=0) yielding no constraint on A or B, Eq. 5.61 holds automatically, and Eq. 5.62 gives

$$kA - (-1)^j k\left[A(-1)^j - 0\right] = \frac{2m\alpha}{\hbar^2}B \Rightarrow B = 0.\quad \text{So}\quad \boxed{\psi = A\sin kx.}$$

Here ψ is *zero* at each delta spike, so the wave function never "feels" the potential at all.

Problem 5.22

(a)

$$\psi(x_A,x_B,x_C) = \frac{1}{\sqrt{6}}\left(\sqrt{\frac{2}{a}}\right)^3\left[\sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) - \sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right)\right.$$

$$+\sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right) - \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right)$$

$$\left.+\sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) - \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right)\right].$$

(b) (i)

$$\psi = \left(\sqrt{\frac{2}{a}}\right)^3\left[\sin\left(\frac{11\pi x_A}{a}\right)\sin\left(\frac{11\pi x_B}{a}\right)\sin\left(\frac{11\pi x_C}{a}\right)\right].$$

(ii)

$$\psi = \frac{1}{\sqrt{3}}\left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{\pi x_A}{a}\right)\sin\left(\frac{\pi x_B}{a}\right)\sin\left(\frac{19\pi x_C}{a}\right) + \sin\left(\frac{\pi x_A}{a}\right)\sin\left(\frac{19\pi x_B}{a}\right)\sin\left(\frac{\pi x_C}{a}\right) + \sin\left(\frac{19\pi x_A}{a}\right)\sin\left(\frac{\pi x_B}{a}\right)\sin\left(\frac{\pi x_C}{a}\right)\right].$$

(iii)

$$\psi = \frac{1}{\sqrt{6}}\left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) + \sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) + \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right) + \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) + \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) + \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right)\right].$$