Problem A.1

(a) Yes; two-dimensional.

(b) No; the sum of two such vectors has $a_z = 2$, and is not in the subset. Also, the null vector (0,0,0) is not in the subset.

(c) Yes; one-dimensional.

Problem A.5

From Eq. A.21: $\langle\gamma|\gamma\rangle = \langle\gamma|\left(|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}|\alpha\rangle\right) = \langle\gamma|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}\langle\gamma|\alpha\rangle.$ From Eq. A.19:

$$\langle\gamma|\beta\rangle^* = \langle\beta|\gamma\rangle = \langle\beta|\left(|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}|\alpha\rangle\right) = \langle\beta|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}\langle\beta|\alpha\rangle = \langle\beta|\beta\rangle - \frac{|\langle\alpha|\beta\rangle|^2}{\langle\alpha|\alpha\rangle},$$ which is *real*.

$$\langle\gamma|\alpha\rangle^* = \langle\alpha|\gamma\rangle = \langle\alpha|\left(|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}|\alpha\rangle\right) = \langle\alpha|\beta\rangle - \frac{\langle\alpha|\beta\rangle}{\langle\alpha|\alpha\rangle}\langle\alpha|\alpha\rangle = 0.$$ $\langle\gamma|\alpha\rangle = 0.$ So (Eq. A.20) :

$$\langle\gamma|\gamma\rangle = \langle\beta|\beta\rangle - \frac{|\langle\alpha|\beta\rangle|^2}{\langle\alpha|\alpha\rangle} \geq 0,$$ and hence $|\langle\alpha|\beta\rangle|^2 \leq \langle\alpha|\alpha\rangle\langle\beta|\beta\rangle.$ QED

Problem A.6

$$\langle\alpha|\beta\rangle = (1-i)(4-i) + (1)(0) + (-i)(2-2i) = 4 - 5i - 1 - 2i - 2 = 1 - 7i; \quad \langle\beta|\alpha\rangle = 1 + 7i;$$

$$\langle\alpha|\alpha\rangle = 1 + 1 + 1 + 1 = 4; \quad \langle\beta|\beta\rangle = 16 + 1 + 4 + 4 = 25; \quad \cos\theta = \sqrt{\frac{1+49}{4\cdot 25}} = \frac{1}{\sqrt{2}}; \quad \boxed{\theta = 45^\circ.}$$