Chapter 07: Kinetic Energy and Work

Like other fundamental concepts, “energy” is harder to define in words than in equations. It is closely linked to the concept of “force”.

Conservation of Energy is one of Nature’s fundamental laws that is not violated.

Energy can take on different forms in a given system.

Forms of energy

Kinetic energy: the energy associated with motion.
Potential energy: the energy associated with position or state. (Chapter 08)
Heat: thermodynamic quanitity related to entropy.

Conservation of Energy is one of Nature's fundamental laws that is not violated.

That means the grand total of all forms of energy in a given system is (and *was*, and *will be*) a constant.

Different forms of energy

Kinetic Energy:
linear motion
rotational motion

Potential Energy:
gravitational
spring compression/tension
electrostatic/magnetostatic
chemical, nuclear, etc....

Mechanical Energy is the sum of Kinetic energy + Potential energy. (reversible process)

Friction will convert mechanical energy to heat. Basically, this (conversion of mechanical energy to heat energy) is a non-reversible process.

Kinetic Energy

- Kinetic Energy is the energy associated with the motion of an object

$$\mathbf{K = (1/2)\, mv^2}$$

m: mass and v: speed

This form of K.E. holds only for speeds $v \ll c$.

- SI unit of energy:

  1 joule = 1 J = 1 $kg{\cdot}m^2/s^2$

- Other useful unit of energy is the electron volt (eV)

  1 eV = 1.602×10^{-19} J

Work

- Work is energy transferred to or from an object by means of a force acting on the object

- Work done by a **constant** force only:

  $W = (F \cos \phi)\ d = F\ d \cos \phi.$

In general when force changes with time,

$$W = \int \vec{F} \cdot d\vec{s}$$

- Work done by a constant force:

$$W = F\,d\cos\phi = \vec{F}\cdot\vec{d}$$

  - when $\phi < 90^o$, W positive
  - when $\phi > 90^o$, W negative
  - when $\phi = 90^o$, $W = 0$
  - When F or d is zero, $W = 0$

  - Work ***done on the object by the force***
  - **Positive work**: object receives energy
  - **Negative work**: object loses energy

- $W = F\, d \cos \phi = \vec{F} \cdot \vec{d}$

- SI Unit for Work

  $1\ \mathrm{N} \cdot \mathrm{m} = 1\ \mathrm{kg} \cdot \mathrm{m/s^2} \cdot \mathrm{m} = 1\ \mathrm{kg} \cdot \mathrm{m^2/s^2} = 1\ \mathbf{J}$

- Net work done by several forces

  $\Sigma W = \vec{F}_{net} \cdot \vec{d} = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3) \cdot \vec{d}$

  $= \vec{F}_1 \cdot \vec{d} + \vec{F}_2 \cdot \vec{d} + \vec{F}_3 \cdot \vec{d} = W_1 + W_2 + W_3$

Consider 1-D motion.

$$W = \int_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} (m\,a)\,dx = \int_{x_i}^{x_f} m\left(\frac{dv}{dt}\right) dx$$

$$= \int_{x_i}^{x_f} m\left(\frac{dv}{dx}\frac{dx}{dt}\right) dx = \int_{x_i}^{x_f} mv\left(\frac{dv}{dx}\right) dx$$

$$= \int_{v_i}^{v_f} mv\,dv = \frac{1}{2}mv^2 \Big|_{v_i}^{v_f} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

So, kinetic energy is mathematically connected to work!!

Work-Kinetic Energy Theorem

The change in the kinetic energy of a particle is equal the net work done on the particle

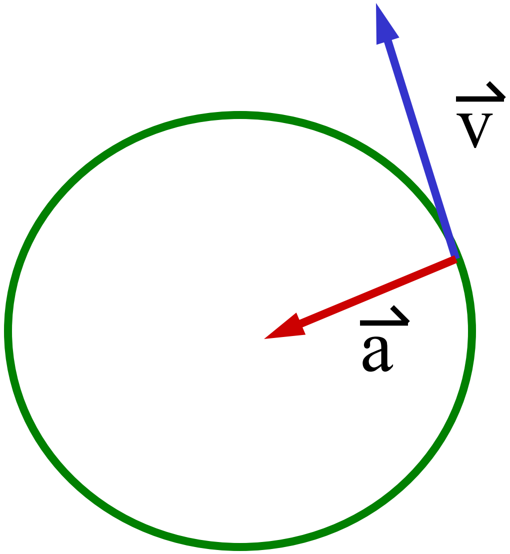
$$\Delta K = K_f - K_i = W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

.... or in other words,

final kinetic energy = initial kinetic energy + net work

$$K_f = \frac{1}{2}mv_f^2 = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net}$$

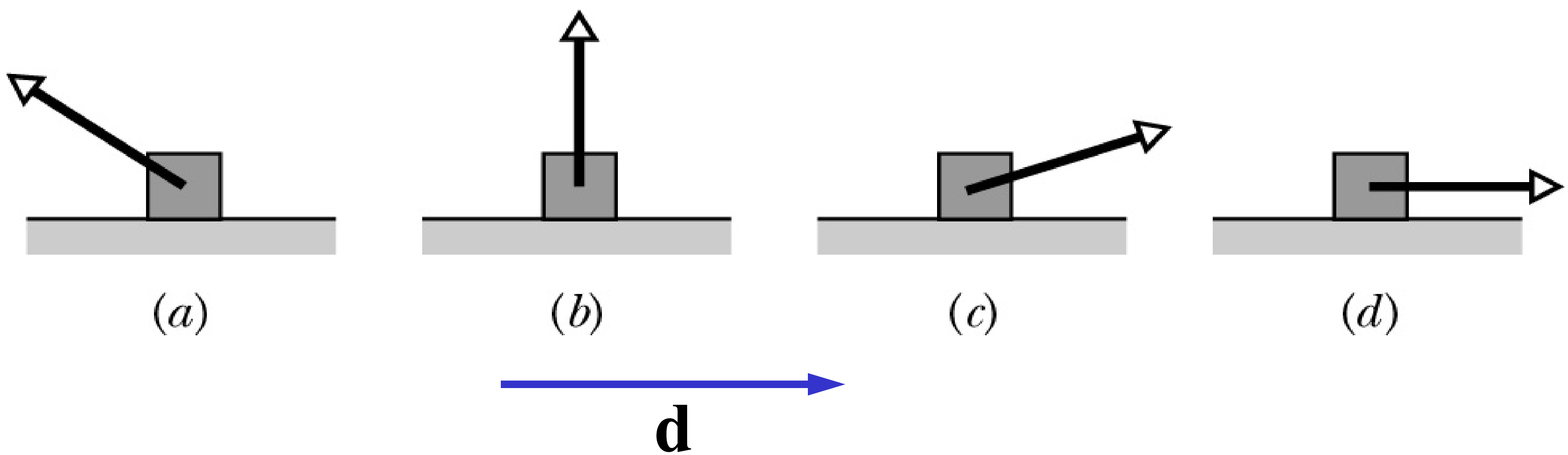
Question: What about circular motion?



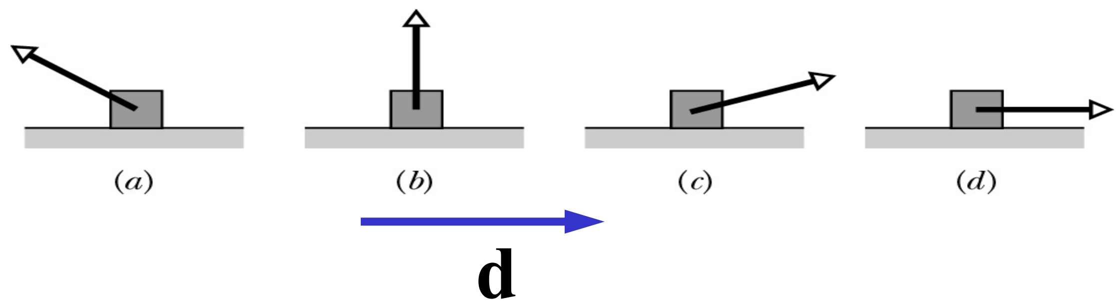
How much work was done and when?

A Quiz

In all of the four situations shown below, the box is sliding to the right a distance of d. The magnitudes of the forces are identical. Rank the work done during the displacement, from most positive to most negative.



A Quiz

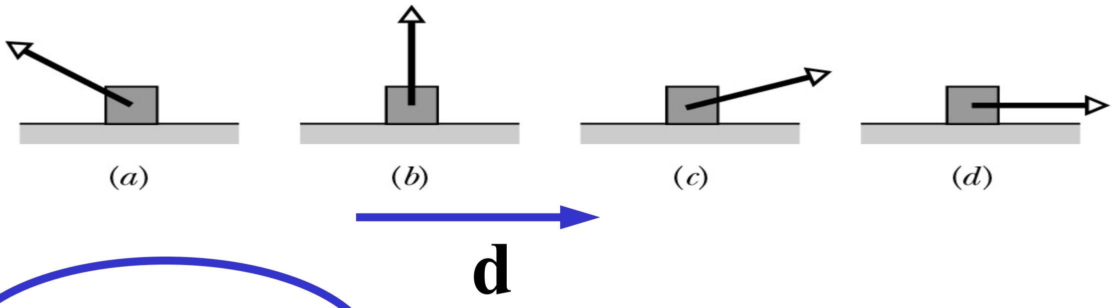


W+ $\longrightarrow$ W−

1) a,b,c,d
2) d,c,b,a
3) b,a,c,d
4) all result in the same work
0) none of the above

A Quiz

$$W = F\,d\cos\phi = \vec{F}\cdot\vec{d}$$



W+ $\longrightarrow$ W–

1) a,b,c,d   2) d,c,b,a   3) b,a,c,d

4) all result in the same work   0) none of the above

A particle moves along the x-axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes:

(a) from –3 m/s to –2 m/s?

$|v_f| < |v_i|$ means $K_f < K_i$, so work is negative

(b) from –2 m/s to 2 m/s?

–2 m/s $\longrightarrow$ 0m/s: work negative

0 m/s $\longrightarrow$ +2m/s: work positive

Together they add to zero work.

(c) In each situation, is the work done positive, negative or zero?

Work done by Gravitation Force

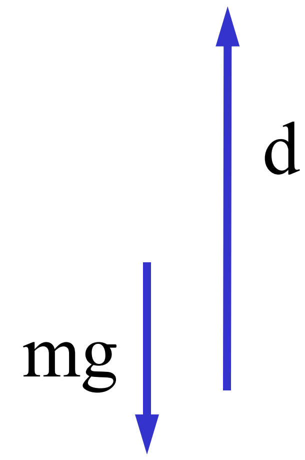
$W_g = mg\, d \cos \phi$

throw a ball upwards:

– during the rise,

$W_g = mg\, d \cos 180^o = -mgd$

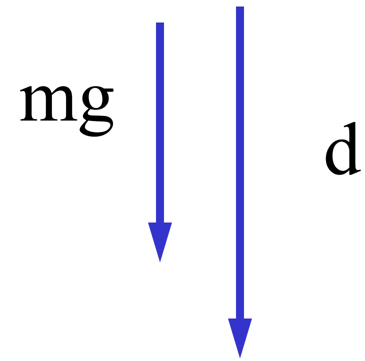
negative work, K, v decrease



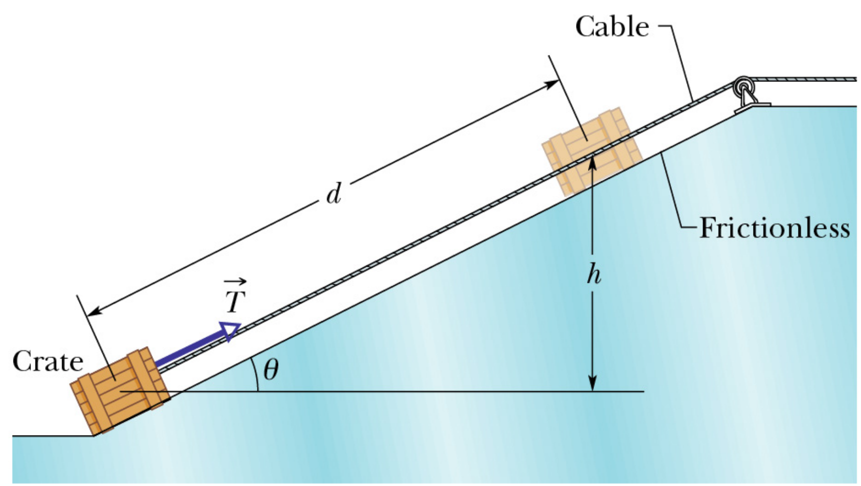
– during the fall,

$W_g = mgd \cos 0^o = mgd$

positive work, K, v increase

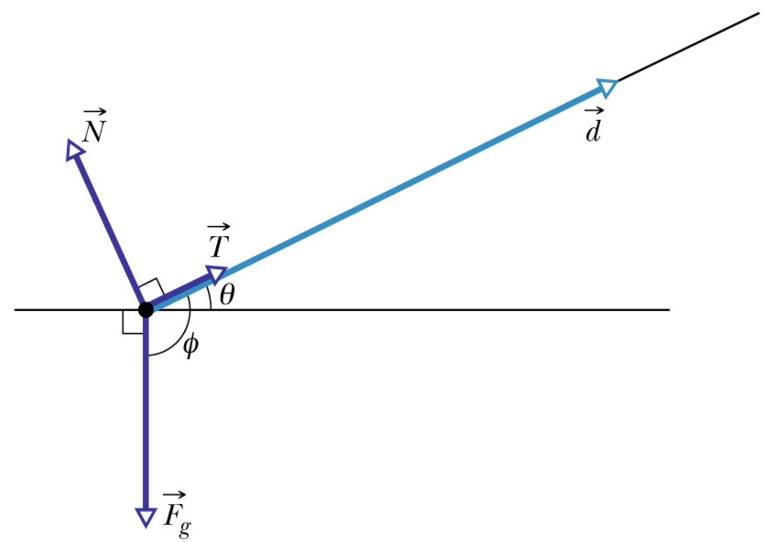


Sample problem 7-5: An initially stationary 15.0 kg crate is pulled a distance L = 5.70 m up a frictionless ramp, to a height H of 2.5 m, where it stops.



(a)

(a) How much work W_g is done on the crate by the gravitational force $\vec{\mathbf{F}}_g$ during the lift?



(b)

$\vec{\mathbf{F}}_g$ is constant in this problem, so we can use $W = \vec{\mathbf{F}}_g \cdot \vec{d} = mgd\cos\phi$, where $\phi = 90^o + \theta$

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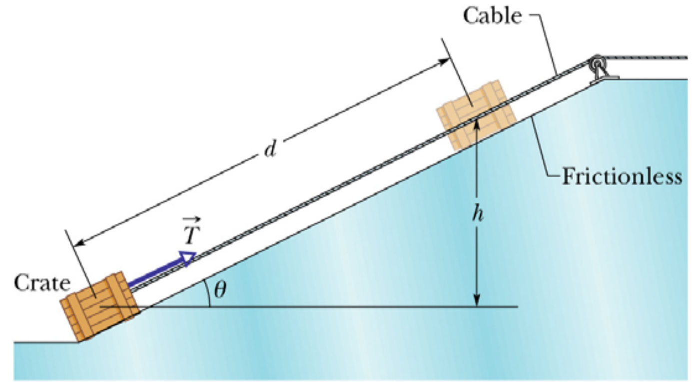
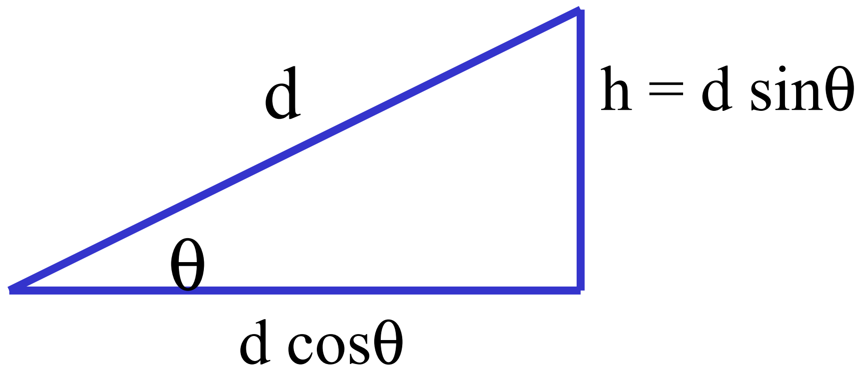
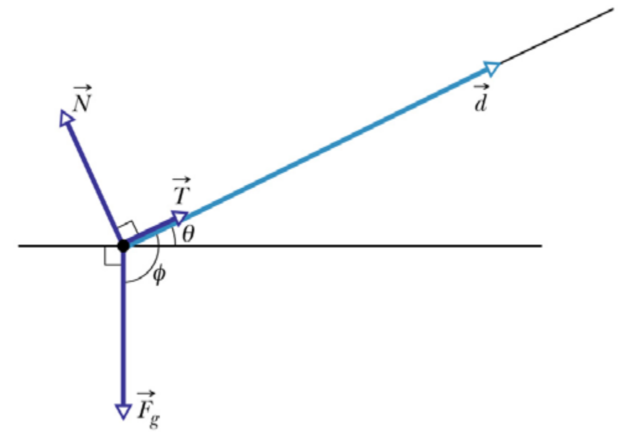


(a)

Notice the triangle that is set up:





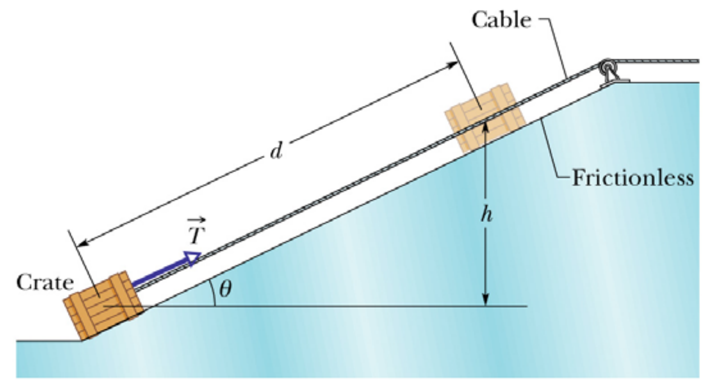
(b)

$$W = \vec{\mathbf{F}}_g \cdot \vec{d} = mgd \cos(90^o + \theta) = -mgd \sin\theta = -mgh = -368 \text{ J}$$

Work done against force of gravity, therefore negative.

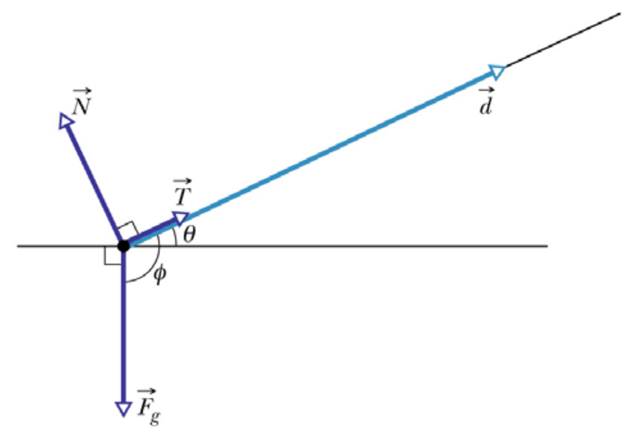
This work is just the negative of the change in potential energy.

How much work W_t is done on the crate by the force **T** from the cable during the lift?



(a)

$v_f = v_i = 0$ means $\Delta K=0$ and the net work must equal zero.



(b)

$$\begin{array}{l} \quad W_g = -368 \text{ J} \\ + W_N = 0 \text{ J} \\ + W_T \\ \hline = 0 \end{array}$$

because $\vec{N} \perp \vec{d}$

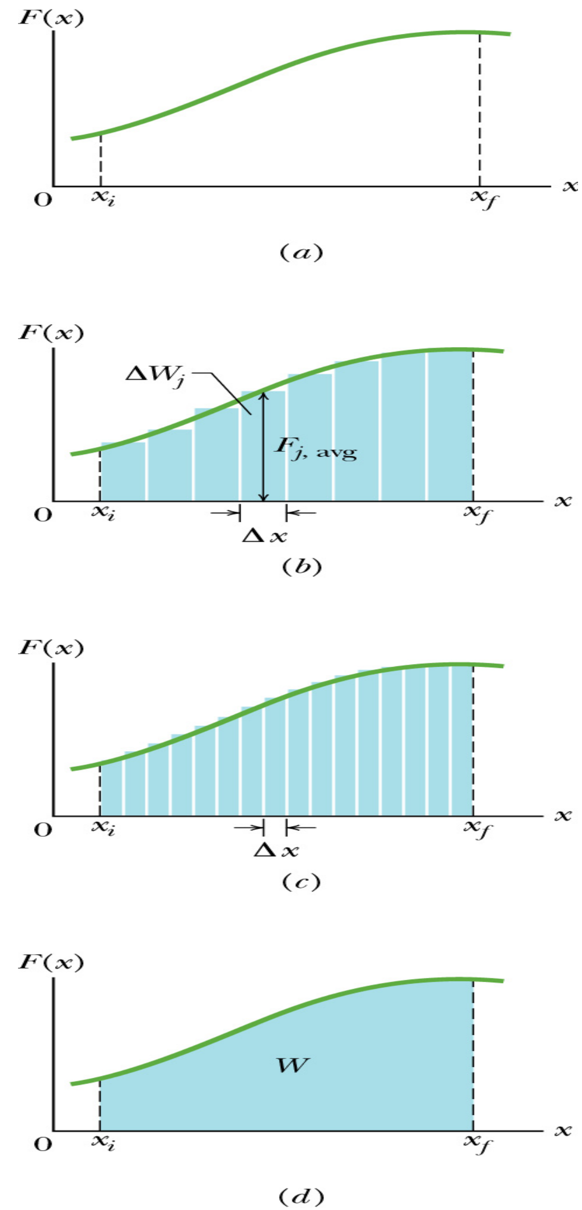
$W_T = +368$ J

Work done by a variable force

$$\Delta W_j = F_{j,\,avg}\Delta x$$

$$W = \Sigma\Delta W_j = \Sigma F_{j,\,avg}\Delta x = \lim_{\Delta x \to 0}\Sigma F_{j,\,avg}\Delta x$$

$$W = \int_{x_i}^{x_f} F(x)dx$$



- Three dimensional analysis

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

$$= \int_{r_i}^{r_f} \vec{F}\cdot d\vec{r}$$

Work done by a spring force

- The spring force is given by

  $\vec{F} = -k\vec{x}$ **(Hooke's law)**

  k: spring (or force) constant

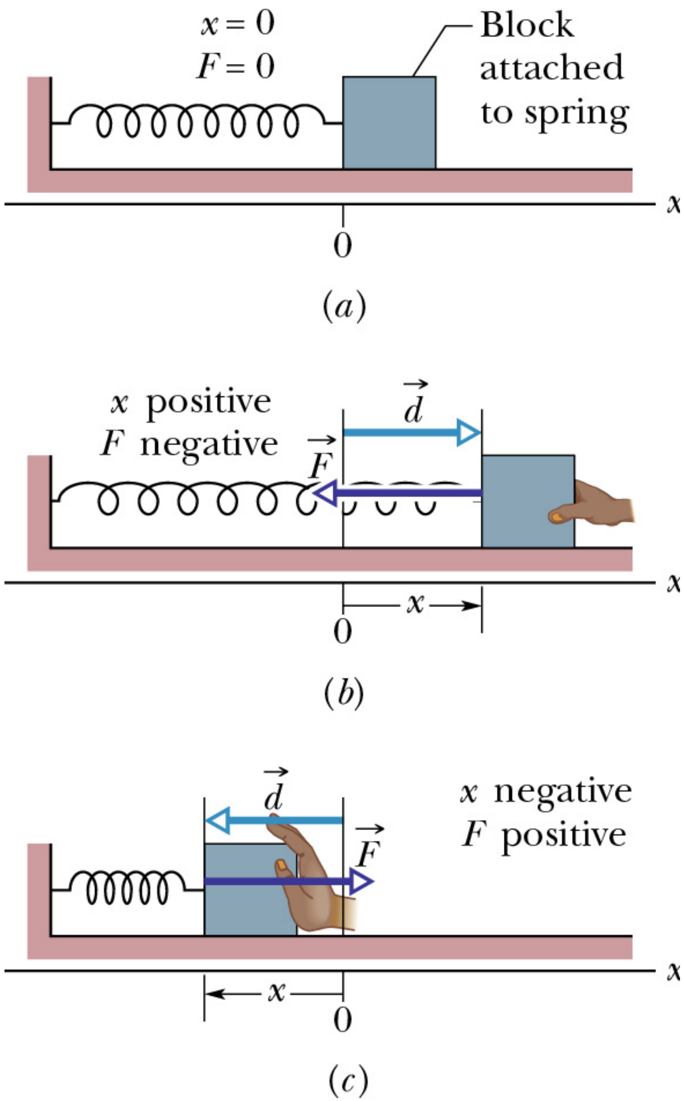
  k relates to stiffness of spring;

  unit for k: N/m.

  Spring force is a variable force

  x = 0 at the free end of

  the relaxed spring.



Work done by a spring force

The spring force tries to restore the system to its equilibrium state (position).

Examples:

- springs
- molecules
- pendulum

Motion results in Simple Harmonic Motion

Work done by Spring Force

- Work done *by* the spring force

$$W_s = \int_{x_i}^{x_f} F dx = \int_{x_i}^{x_f} (-kx) dx = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

- If $|x_f| > |x_i|$ (further away from $x = 0$); $W < 0$

  If $|x_f| < |x_i|$ (closer to $x = 0$); $W > 0$

- If $x_i = 0$, $x_f = x$ then $W_s = -\frac{1}{2} k x^2$

This work went into potential energy, since the speeds are zero before and after.

- The work done by an applied force (i.e., *us*).
- If the block is stationary both before and after the displacement:

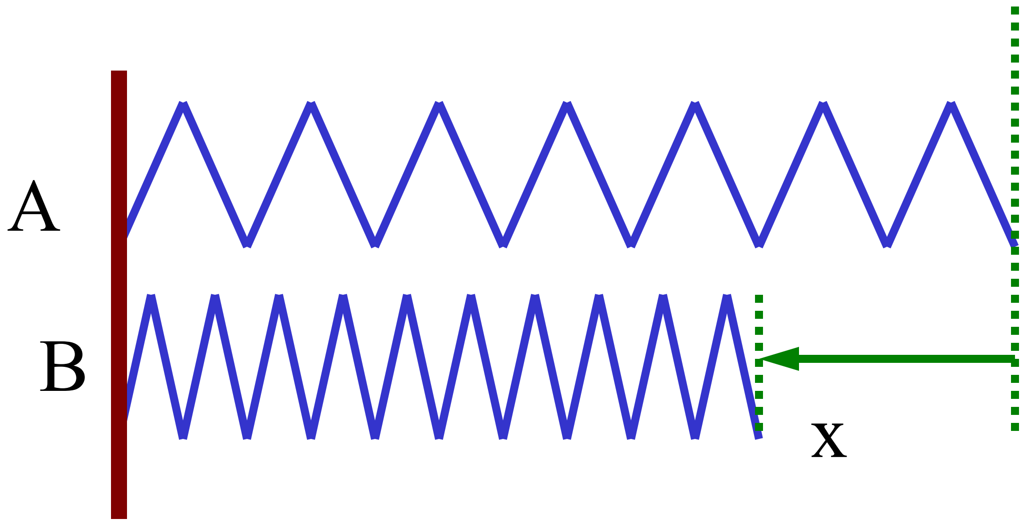
  $v_i = v_f = 0$ then $K_i = K_f = 0$

  work-kinetic energy theorem: $W_a + W_s = \Delta K = 0$

  therefore: $W_a = -W_s$

A Quiz

We decide to compress a spring a distance x.



Did we do positive, negative, or no work on the spring?

1) we did positive work
2) we did negative work
3) we didn't do any work
0) none of the above

A Quiz

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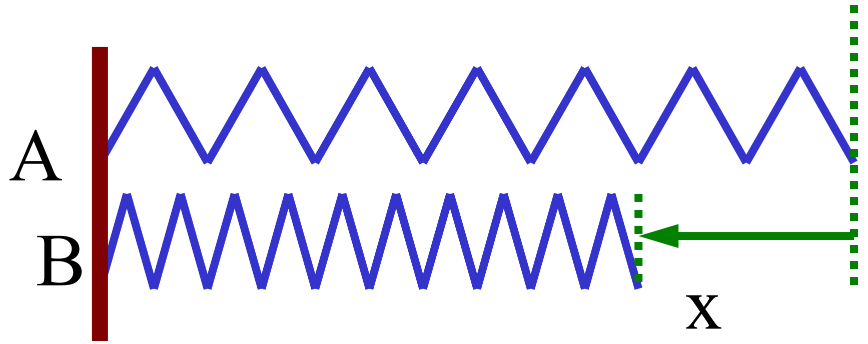
Did we do positive, negative, or no work on the spring?



1) we did positive work
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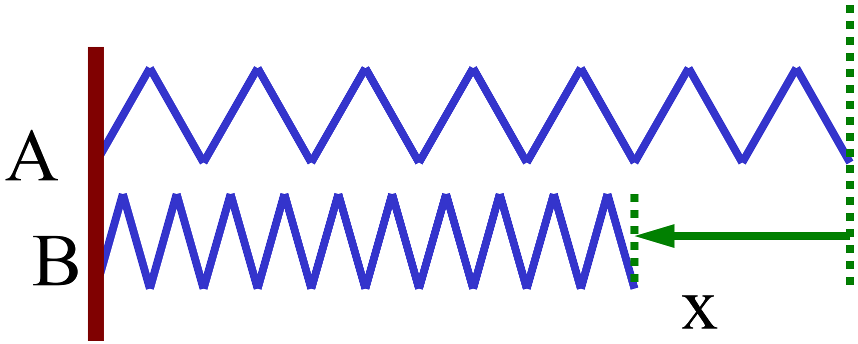
A Quiz

We decide to compress a spring a distance x.

$F_s = -kx$ is the spring force we had to counteract with $F_{applied} = kx$. Thus, our work $W = 1/2kx^2$ is positive.

Did we do positive, negative, or no work on the spring?



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Power

- The time rate at which work is done by a force
- Average power

  $P_{avg} = W/\Delta t$
- Instantaneous power

  $P = dW/dt = (F\cos\phi\ dx)/dt = F\ v\ \cos\phi = \vec{F} \cdot \vec{v}$

  Unit: watt

  1 watt = 1 W = 1 J / s

  1 horsepower = 1 hp = 550 ft lb/s = 746 W
- kilowatt-hour is a unit for energy or work:

  1 kW h = 3.6 M J