Chapter 5  Force and Motion

• **Force** $\mathbf{F}$
  - is the interaction between objects
  - is a vector
  - causes acceleration
  - **Net force**: vector sum of all the forces on an object.

\[
\mathbf{F}_{\text{total}} \equiv \mathbf{F}_{\text{net}} = \sum_{i=1}^{N} \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \ldots = m\mathbf{a}
\]
Force Examples

- Gravitational
- Friction
- tension
- spring
- normal

- momentum change
- electrostatic
- magnetic
- nuclear

- etc.......
Newton’s first law

• **Newton’s first law**: If no force acts on a body, then the body’s velocity cannot change, that is, the body can not accelerate
  – rest, still rest
  – moving, continue moving with same velocity

• Inertia reference frame is one in which Newton’s laws hold
Mass

- **Definition:** The mass of an object is a measure of its “resistance” to being accelerated.
- Symbol: $m$
- SI Base unit: kg (by the way, the English unit for mass is the “slug”)
- Scalar quantity
- Mass is an intrinsic characteristic of an object, however its value does change at high speeds (Special Relativity) Has to do with the speed of light being $c$. 
Newton’s Second Law

- **Newton’s second law**: The net force on a body is equal to the product of the body’s mass and the acceleration of the body
  \[ \sum \vec{F} = m \vec{a} \]

  \( \sum \vec{F} \): vector sum of all the forces that act on that body

  \[ \sum F_x = m a_x \quad \sum F_y = m a_y \quad \sum F_z = m a_z \]

- **Unit**: \( 1 \text{ N} = (1 \text{ kg}) \cdot (1 \text{ m/s}^2) = 1 \text{ kg.m/s}^2 \)
Example: (Problem 5-6 in the text book) Three astronauts, propelled by jet backpacks, push and guide a 120 kg asteroid toward a processing dock, exerting forces shown in Fig. What is the asteroid’s acceleration (a) in unit vector notation and as (b) a magnitude and (c) a direction.

Break problem into x- and y-components

\[
\vec{F}_1 = (41 \text{N} \cos 60^\circ) \hat{i} + (-41 \text{N} \sin 60^\circ) \hat{j} \\
\vec{F}_2 = (55 \text{N}) \hat{i} + (0) \hat{j} \\
\vec{F}_3 = (32 \text{N} \cos 30^\circ) \hat{i} + (32 \text{N} \sin 30^\circ) \hat{j}
\]

\[
\vec{F}_{\text{net}} = (103.2 \text{N}) \hat{i} + (-19.51 \text{N}) \hat{j}
\]
• The gravitational force near the surface of a very large object (i.e., the Earth):

\[ F_g = m \, g \]

• Weight (gravitational force)

\[ W = F_g = m \, g \]

- \( g \) varies with location

- **weight and mass are different**, e.g. 7.2 kg ball, same mass on earth and moon, weight 71 N on earth but 12 N on the moon
Suppose you are talking by interplanetary telephone to your friend, who lives on the moon. She tells you that he just won a piece of gold weighing one newton in a contest. Excitedly, you tell her that you entered the Earth version of the same contest and also won a newton of gold! Who is richer?
• The normal force: $F_N$
  – When a body presses against a surface, the surface deforms and pushes on the body with a normal force $N$ that is \textit{perpendicular} to the surface

  – $F_N$ does not always equal $mg$

• Friction: $f$
  – the resistance force on a body when the body slides or attempts to slide along a surface
• Tension: $T$
  – When a cord is attached to a body and pulled taut, the cord pulls on the body with force $T$

CP 5-5: The body in fig (c) has a weight of 75 N, Is $T$ equal to, greater than, or less than 75 N when the body is moving upward (a) at constant speed (b) at increasing speed (c) at decreasing speed?
Newton’s Third Law

- Why would I feel pain if I hit the wall with my fist?
- Newton’s third law: When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction

\[ F_{AB} = - F_{BA} \]

- They do not cancel each other since they act on different bodies
If a sport car collides head-on with a massive truck,
(a) which vehicle experiences the greater force?
(b) which vehicle experience the greater acceleration?

Remember that a modern semi-truck has a mass of 25 cars!
Assume \( F = 20 \text{ N} \), surface frictionless. What is the acceleration?

\[
F = (m_1 + m_2 + m_3 + m_4)a = (20 \text{ kg})a = 20 \text{ N}
\]

\[\Rightarrow a = \frac{20 \text{ N}}{20 \text{ kg}} = 1.0 \text{ m/s}^2\]

How about \( T_1 \)?

\[
T_1 = m_1a = (10 \text{ kg})(1.0 \text{ m/s}^2) = 10 \text{ N}
\]
General scheme for solving Newton’s law problems

- Isolate the objects in the problem
- For each object of interest, identify all the external forces on that object and draw a free-body diagram for this object
- Establish a convenient coordinate system for each object and find the component of the forces along those axes.
- Apply Newton’s 2\textsuperscript{nd} law in the x and y directions for each object. (e.g. \( \Sigma F_x = ma_x \), \( \Sigma F_y = ma_y \))
- Solve the resulting set of equations.
A Quiz

Consider a traffic light of \( m = 4 \text{kg} \) held by one rope which in turn is supported by two other ropes as shown with angles \( \theta_1 = 30^\circ \theta_2 = 45^\circ \), Which of the three ropes has the greater tension?
Consider a traffic light of $m = 4\text{kg}$ held by one rope which in turn is supported by two other ropes as shown with angles $\theta_1 = 30^\circ$, $\theta_2 = 45^\circ$. Which of the three ropes has the greater tension?

1) rope 1  
2) rope 2  
3) rope 3  
4) All ropes have the same tension
Consider a traffic light of $m = 4$ kg held by one rope which in turn is supported by two other ropes as shown with angles $\theta_1 = 30^\circ$, $\theta_2 = 45^\circ$, Which of the three ropes has the greater tension?

1) rope 1  
2) rope 2  
3) rope 3  
4) All ropes have the same tension
A Quiz

Consider a traffic light of \( m = 4 \text{kg} \) held by one rope which in turn is supported by two other ropes as shown with angles \( \theta_1 = 30^\circ \), \( \theta_2 = 45^\circ \). Which of the three ropes has the greater tension?

\[
F_x = -F_1 \cos 30^\circ + F_2 \cos 45^\circ = 0
\]

\[
F_y = F_1 \sin 30^\circ + F_2 \sin 45^\circ - F_3 = 0
\]

\[
F_1 \cos 30^\circ = F_2 \cos 45^\circ
\]

\[
F_1 \sin 30^\circ + F_2 \sin 45^\circ = F_3
\]

\[
F_1 = 0.732 F_3
\]

\[
F_2 = 0.896 F_3
\]

\[
F_3 = 1.00 F_3
\]
Newton’s Laws of Motion

- **Newton’s first law:** If no force acts on a body, then the body’s velocity cannot change, that is, the body can not accelerate.

- **Newton’s second law:** The net force on a body is equal to the product of the body’s mass and the acceleration of the body \( \Sigma F = m a \)

- **Newton’s third law:** When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction

\[
F_{AB} = - F_{BA}
\]
Sample problem 5-5.
M = 3.3 kg, m = 2.1 kg, frictionless surface
H falls as S accelerate to the right
(a) What is the acceleration of S?

Acceleration a links the two masses together

Forces on m

\[ mg - T = ma \]

unbalanced forces on M

\[ T = Ma \]

\[ mg - Ma = ma \]

\[ a = \frac{m}{M + m} g \]
Consider \( m_1 > m_2 \). What is the acceleration of either mass if the inclined plane is frictionless?

How do you set up this problem? Which Coordinate system?

Easiest because of direction of motion
Consider \( m_1 > m_2 \). What is the acceleration of either mass if the inclined plane is frictionless?

First set up coordinate system
Consider $m_1 > m_2$. What is the acceleration of either mass if the inclined plane is frictionless?

Gravity still points down
Consider $m_1 > m_2$. What is the acceleration of either mass if the inclined plane is frictionless?

- Gravity still points down $m_1g, m_2g$
- Normal force is perpendicular to the plane
- Tension is along the rope
Consider $m_1 > m_2$. What is the acceleration of either mass if the inclined plane is frictionless?

Thus, the free body diagram becomes:

Dissect the forces into components

$$\vec{T} = T\hat{i} + 0\hat{j}$$
$$\vec{N} = 0\hat{i} + N\hat{j}$$
$$m_1\vec{g} = m_1g \sin \theta(-\hat{i}) + m_1g \cos \theta(-\hat{j})$$
Consider \( m_1 > m_2 \). What is the acceleration of either mass if the inclined plane is frictionless?

Thus, the free body diagram becomes:

Disect the forces into components

\[
\begin{align*}
\vec{T} &= T\hat{i} + 0\hat{j} \\
\vec{N} &= 0\hat{i} + N\hat{j} \\
m_1\vec{g} &= m_1g \sin \theta(-\hat{i}) + m_1g \cos \theta(-\hat{j})
\end{align*}
\]
Consider \( m_1 > m_2 \). What is the acceleration of either mass if the inclined plane is frictionless?

Thus, the free body diagram becomes:

Now apply Newton’s Second Law

\[
x : \quad \sum F_{xi} = T - m_1 g \sin \theta = m_1 a \\
y : \quad \sum F_{yi} = N - m_1 g \cos \theta = 0 \\
m_2 : \quad -T + m_2 g = m_2 a
\]
Consider \( m_1 > m_2 \). What is the acceleration of either mass if the inclined plane is frictionless?

Thus, the free body diagram becomes:

Now apply Newton’s Second Law

\[
\begin{align*}
x : \quad \sum F_{xi} &= m_2 g - m_2 a - m_1 g \sin \theta = m_1 a \\
&= m_2 g - m_1 g \sin \theta = (m_1 + m_2) a \\
a &= \frac{m_2 g - m_1 g \sin \theta}{(m_1 + m_2)} \\
&\Rightarrow \quad \ddot{a} = \frac{m_2 g - m_1 g \sin \theta}{(m_1 + m_2)} \hat{i} \\
y : \quad N &= m_1 g \cos \theta
\end{align*}
\]
A Quiz

Consider $m_1 > m_2$. Is the tension in the rope greater than, less than, or equal to $m_2g$?

1) less than $m_2g$  
2) greater than $m_2g$  
3) equal to $m_2g$  
4) depends on the acceleration  
0) none of the above
Consider $m_1 > m_2$. Is the tension in the rope greater than, less than, or equal to $m_2g$?

1) less than $m_2g$  2) greater than $m_2g$  3) equal to $m_2g$
4) depends on the acceleration  0) none of the above
A Quiz

Consider $m_1 > m_2$. Is the tension in the rope greater than, less than, or equal to $m_2g$?

4) depends on the acceleration