Chapter 16 Waves

Types of waves

– Mechanical waves

exist only within a material medium. e.g. water waves, sound waves, etc.

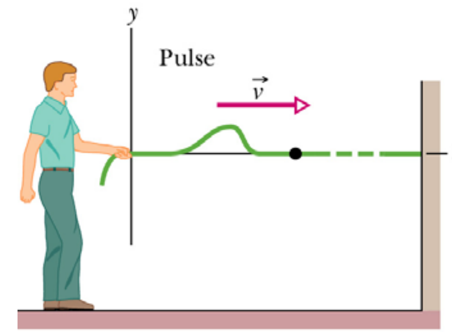
– Electromagnetic waves

require no material medium to exist. e.g. light, radio, microwaves, etc.

– Matter waves

waves associated with electrons, protons, etc.

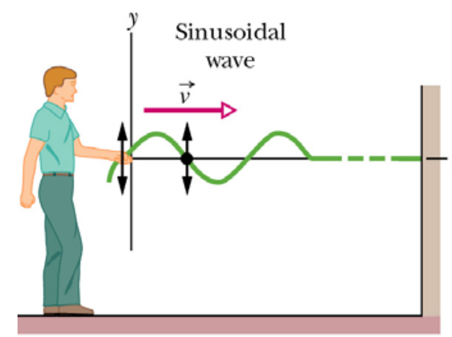
Transverse and Longitudinal Waves



(a)

Transverse waves

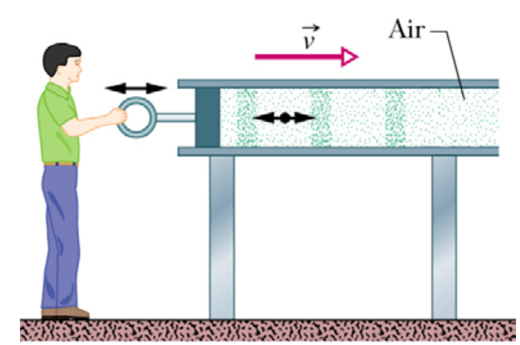
Displacement of every oscillating element is perpendicular to the direction of travel (light)



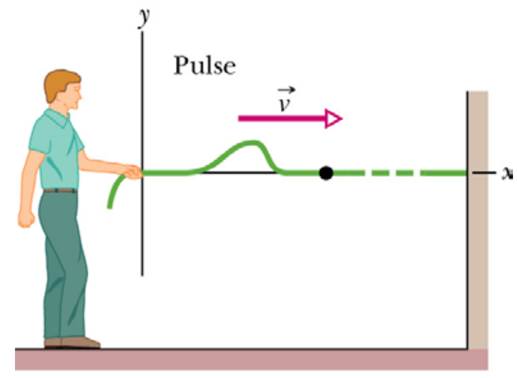
(b)

Longitudinal waves

Displacement of every oscillating element is parallel to the direction of travel (sound)



Describing Waves

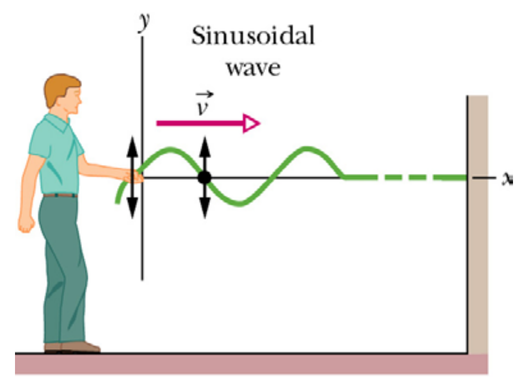


(a)

Remember $\sin\theta$. Let $\theta = kx - \omega t$

For a sinusoidal wave, the displacement of an element located at position x at time t is given by

$$y(x, t) = y_m \sin(kx - \omega t)$$



(b)

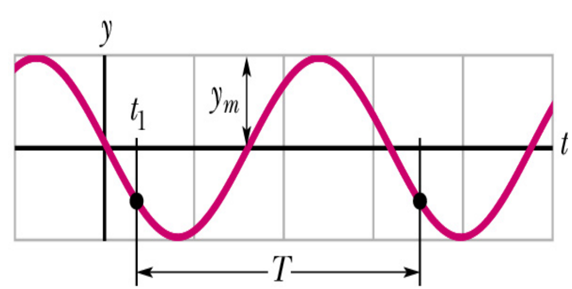
amplitude: y_m

Phase: $(kx - \omega t)$

At a fixed time, $t = t_0$,

$y(x, t_0) = y_m \sin(kx + \text{constant})$

sinusoidal wave form.



At a fixed location, $x = x_0$,

$y(x_0, t) = -y_m \sin(\omega t + \text{constant})$, SHM

Wavelength λ: the distance between repetitions of the wave shape.

$$y(x, t) = y_m \sin(kx - \omega t)$$

at a moment $t = t_0$, $y(x) = y(x + \lambda)$

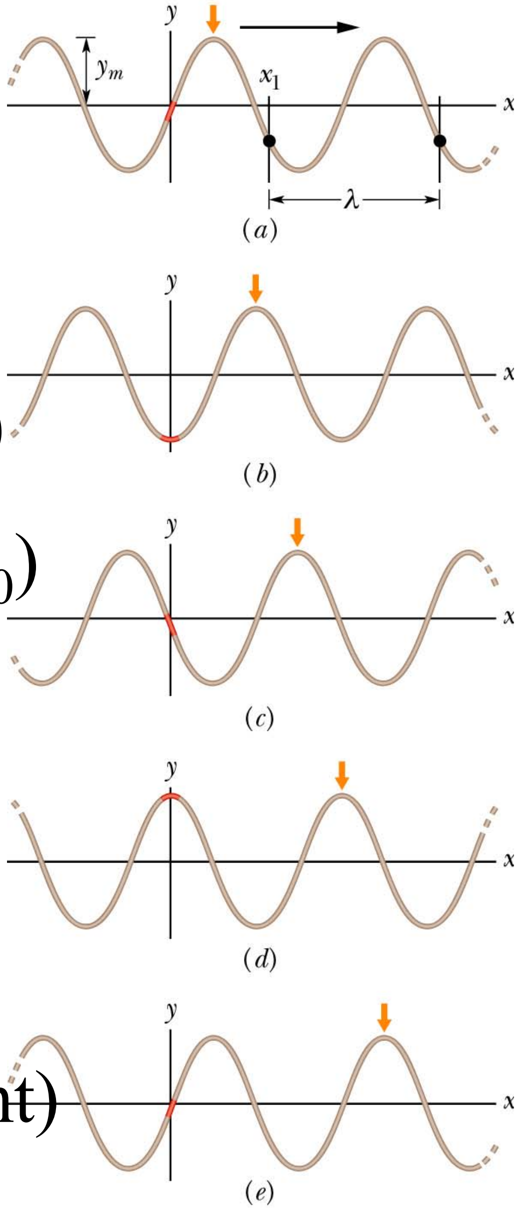
$$y_m \sin(kx - \omega t_0) = y_m \sin(kx + k\lambda - \omega t_0)$$

thus: $k\lambda = 2\pi$

$$k = 2\pi/\lambda$$

k is called angular wave number.

(Note: here k is not spring constant)



- **Period T** : the time that an element takes to move through one full oscillation.

  $y(x, t) = y_m \sin(kx - \omega t)$

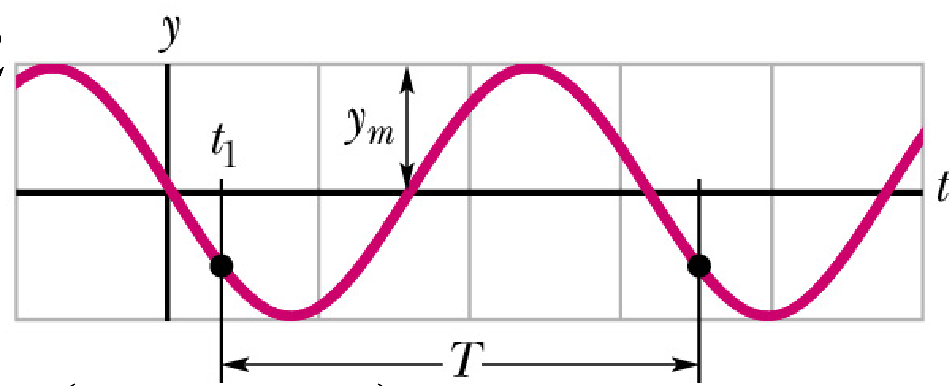
  For an element at $x = x_0$, $y(t) = y(t + T)$

  therefore: $y_m \sin(kx_0 - \omega t) = y_m \sin(kx_0 - \omega(t + T))$

  Thus: $\omega T = 2\pi$

  $\omega = 2\pi/T$ (Angular frequency)

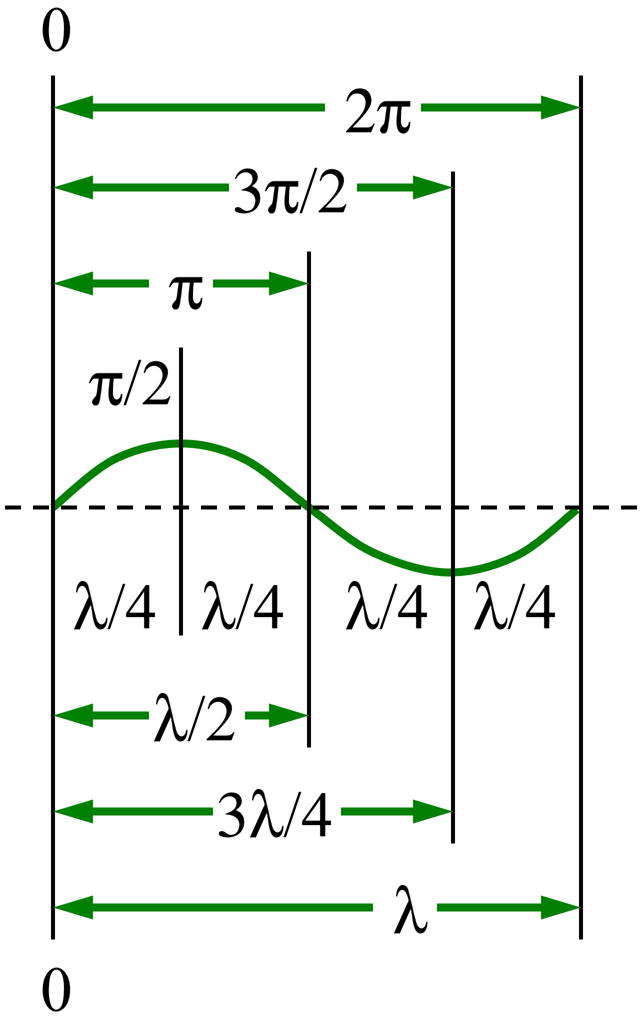
  Frequency: $f = 1/T = \omega /2$



$$y_m \sin(kx - \omega t)$$

$$= y_m \sin(\frac{2\pi}{\lambda} x - 2\pi f t) = y_m \sin\left(2\pi(\frac{x}{\lambda} - f t)\right)$$

Wavelengths and phases



The speed of a traveling wave

- For the wave :

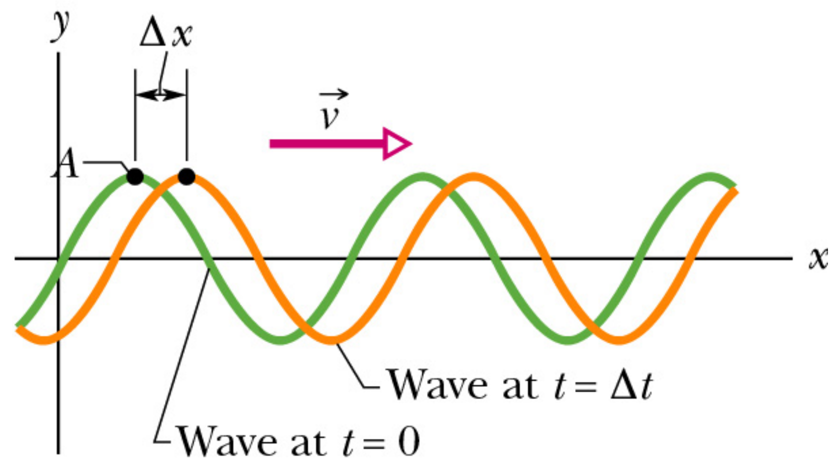
  $y(x, t) = y_m \sin(kx - \omega t)$

  it travels in the positive x direction

  the wave speed:

  $$v = \omega/k$$

  since $\omega = 2\pi/T$, $k = 2\pi/\lambda$ so: $v = \lambda/T = \lambda f$



- $y(x, t) = y_m \sin(kx + \omega t)$

  wave traveling in the **negative** x direction.

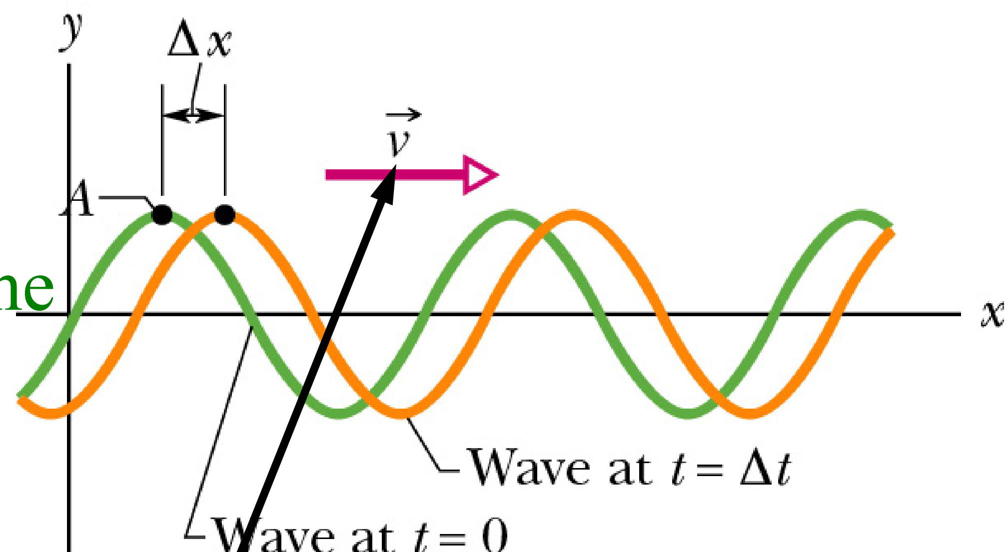
Relationships

General equation of a traveling wave:

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

Constant phase:

$$kx - \omega t + \phi = \text{constant}$$



Derivative of phase w.r.t. time

$$\frac{d(kx - \omega t + \phi)}{dt} = \frac{d(\text{constant})}{dt} = 0$$

$$= k\frac{d(x)}{dt} - \omega\frac{d(t)}{dt} = 0 \Rightarrow \frac{d(x)}{dt} \equiv v_{\text{wave}} = \frac{\omega}{k}$$

$$v = \omega/k$$

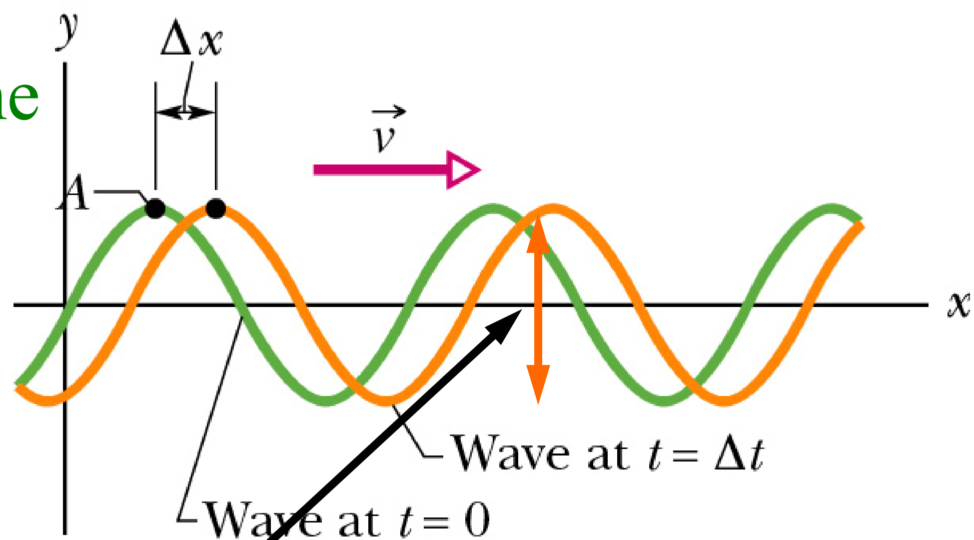
or in words: How fast the wave is traveling in space.

Relationships

General equation of a traveling wave:

$$y(x, t) = y_m \sin(kx - \omega t + \phi)$$

Derivative of y(x, t) w.r.t. time



$$u = v_{particle} \equiv \frac{\partial y(x,t)}{\partial t}$$

$$= \frac{\partial\left(y_m \sin(kx - \omega t + \phi)\right)}{\partial t}$$

$$= -\omega y_m \cos(kx - \omega t + \phi)$$

$$u = v_{particle} \equiv \frac{\partial y(x,t)}{\partial t} = -\omega y_m \cos(kx - \omega t + \phi)$$

or in words: How fast the particle is moving up & down.

A wave traveling along a string is described by

$$y(x, t) = 0.00327\sin(72.1x - 2.72t)$$

where x, y are in m and t is in s.

A) What is the amplitude of this wave?

$y_m = 0.00327\text{m}$

B) What are wavelength and period of this wave?

$k = 2\pi/\lambda$

=> $\lambda = 2\pi/72.1 = 0.0871\text{m}$

$\omega = 2\pi/T = 2.72$

=> $T = 2\pi/\omega = 2.31\text{s}$

A wave traveling along a string is described by

$$y(x, t) = 0.00327\sin(72.1x - 2.72t)$$

where x, y are in m and t is in s.

C) What is velocity of this wave?

$$v = \omega/k = 2.72/72.1 = 0.0377\text{m/s}$$

D) What is the displacement y at x = 0.225m and t = 18.9s?

$$y(x, t) = 0.00327\sin(72.1(0.225) - 2.72(18.9))$$

$$= 0.00327\sin(-35.2\text{rad}) = 0.00327(0.588) = 0.00192\text{m}$$

E) What is the transverse velocity, u, at the same x, t as in (D)?

$$u = \frac{\partial y(x, t)}{\partial t} = \frac{\partial y_m \sin(kx - \omega t)}{\partial t} = -\omega -_m \cos(kx - \omega t)$$

$$= -0.00327(2.72)\cos(-35.2\text{rad}) = -0.00720\text{m/s}$$

Wave speed on a stretched string

Wave speed depends on the medium

For a wave traveling along a stretched string

$$v = \sqrt{\frac{\tau}{\mu}}$$

τ is the tension in the string

μ is the linear density of the string: $\mu = m/l$

v depends on the property (τ and μ) of the string, not on the frequency f. f is determined by the source that generates the wave. λ is then determined by f and v,

$\lambda = v/f$ or in other words,

$$v = f\lambda$$

Energy and power of a traveling string wave

- The oscillating elements have both kinetic energy and potential energy. The average rate at which the energy is transmitted by the traveling wave is:

$P_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2$ (average power)

μ and v depend on the material and tension of the string.

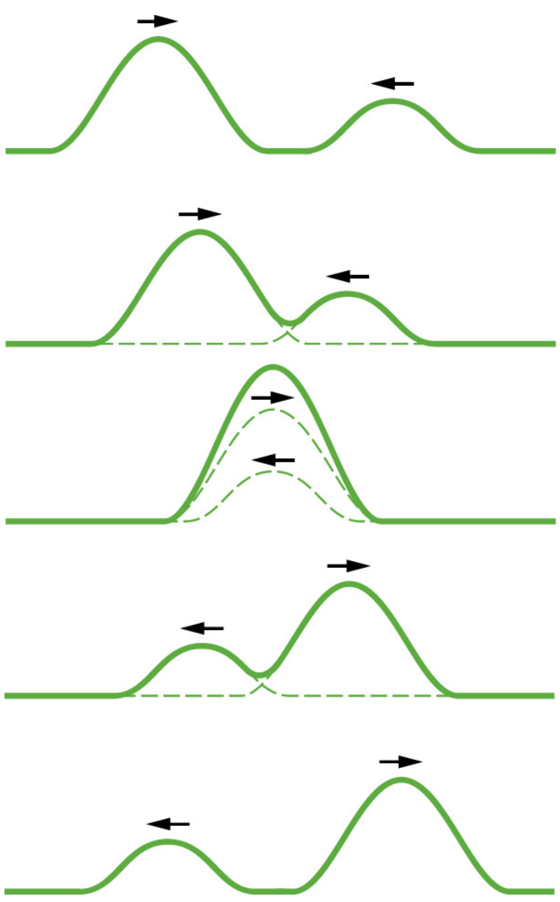
ω and y_m depend on the process that generates the wave.

Principle of Superposition for Waves

- Two waves $y_1(x, t)$ and $y_2(x, t)$ travel simultaneously along the same stretched string, the resultant wave is

$$y(x, t) = y_1(x, t) + y_2(x, t)$$

sum of the displacement from each wave.



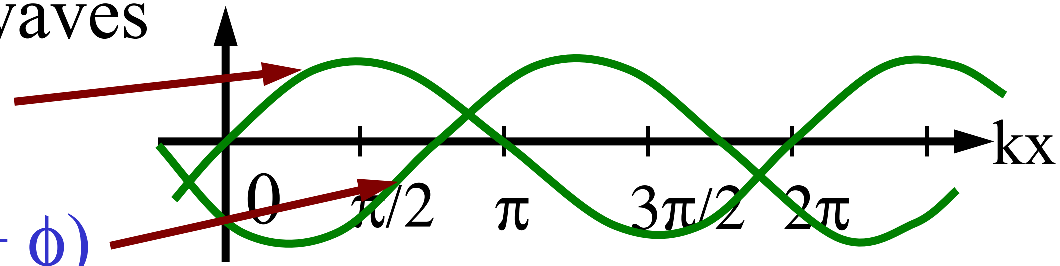
- Overlapping waves do not alter the travel of each other.

Interference of waves

Two equal amplitude waves

$y_1(x,t) = y_m \sin(kx - \omega t)$

$y_2(x,t) = y_m \sin(kx - \omega t + \phi)$



φ: phase difference

Resultant wave:

$$y'(x, t) = y_m (\sin(kx - \omega t) + \sin(kx - \omega t + \phi))$$

Note: $\sin\alpha + \sin\beta = 2\sin[\frac{1}{2}(\alpha + \beta)] \cos[\frac{1}{2}(\alpha - \beta)]$

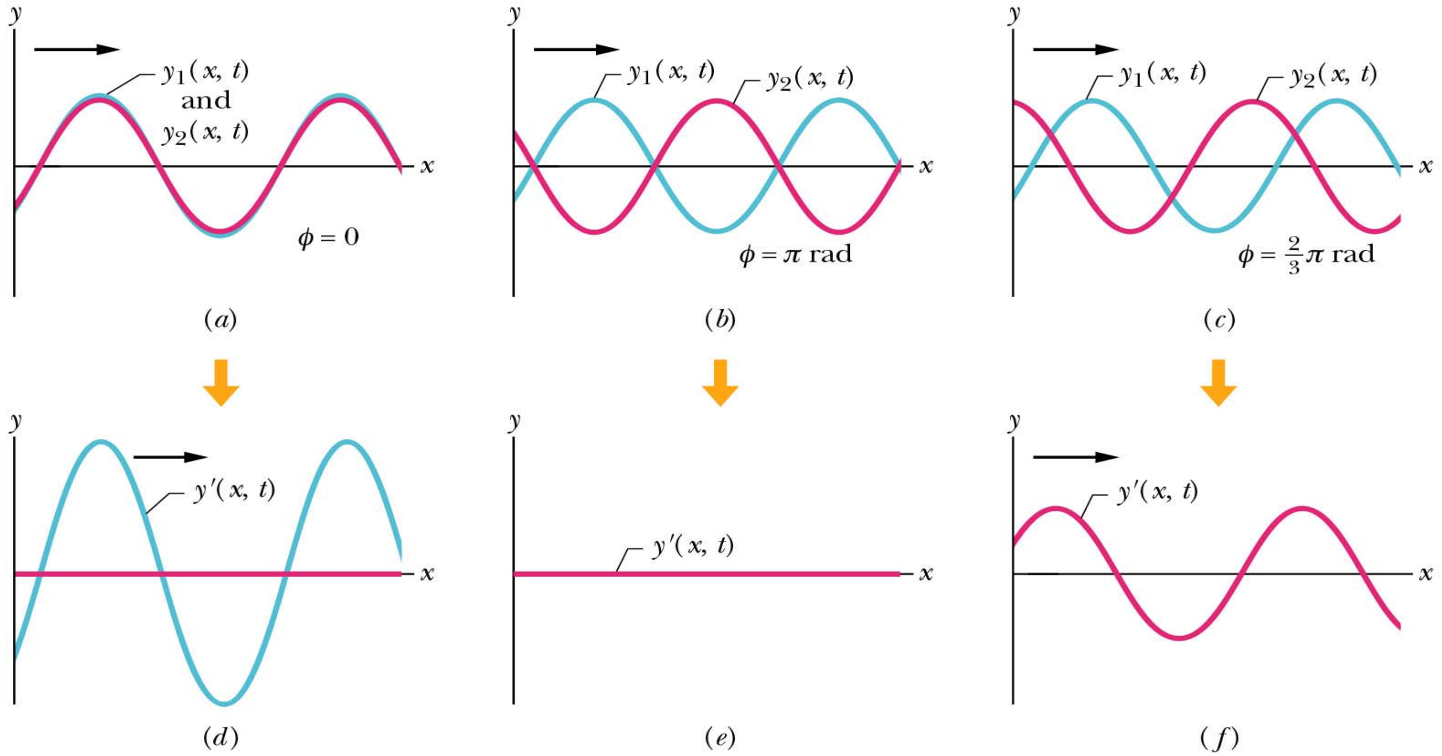
=> $y'(x, t) = [2y_m \cos \frac{1}{2} \phi]\sin(kx - wt + \frac{1}{2} \phi)$

The resultant wave of two interfering sinusoidal waves with same frequency and same amplitude is again another sinusoidal wave with an amplitude of $y'_m = 2y_m \cos \frac{1}{2} \phi$

- $y'_m = 2y_m \cos \frac{1}{2} \phi$
- If $\phi = 0$, i.e. two waves are exactly in phase

  $y'_m = 2y_m$ (fully constructive)
- If $\phi = \pi$ or 180^o, i.e. two waves are exactly out of phase

  $y'_m = 0$ (fully destructive interference)
- For any other values of ϕ, intermediate interference



Phasors

- We can represent a wave with a phasor.

  (no, not the Star Trek kind....)

- Phasor is a vector

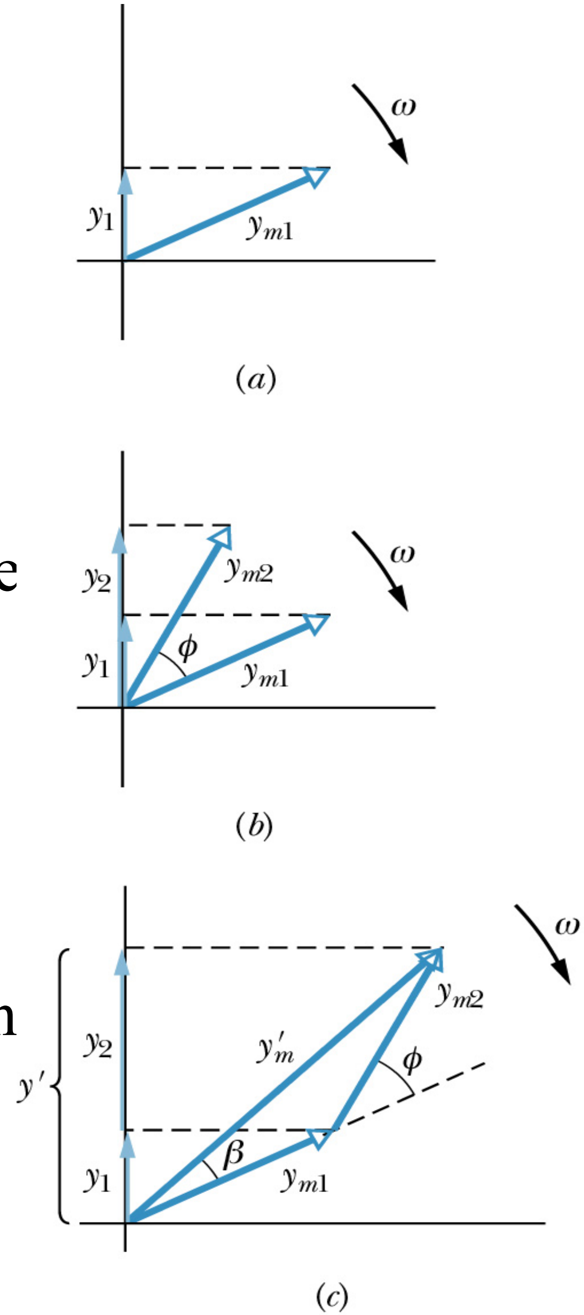
  its magnitude = amplitude of the wave

  its angular speed = angular frequency of the wave.

- Its projection on y axis:

  $y_1(x, t) = y_{m1}\sin(kx - \omega t)$

- We can use phasors to combine waves even if their amplitudes are different.



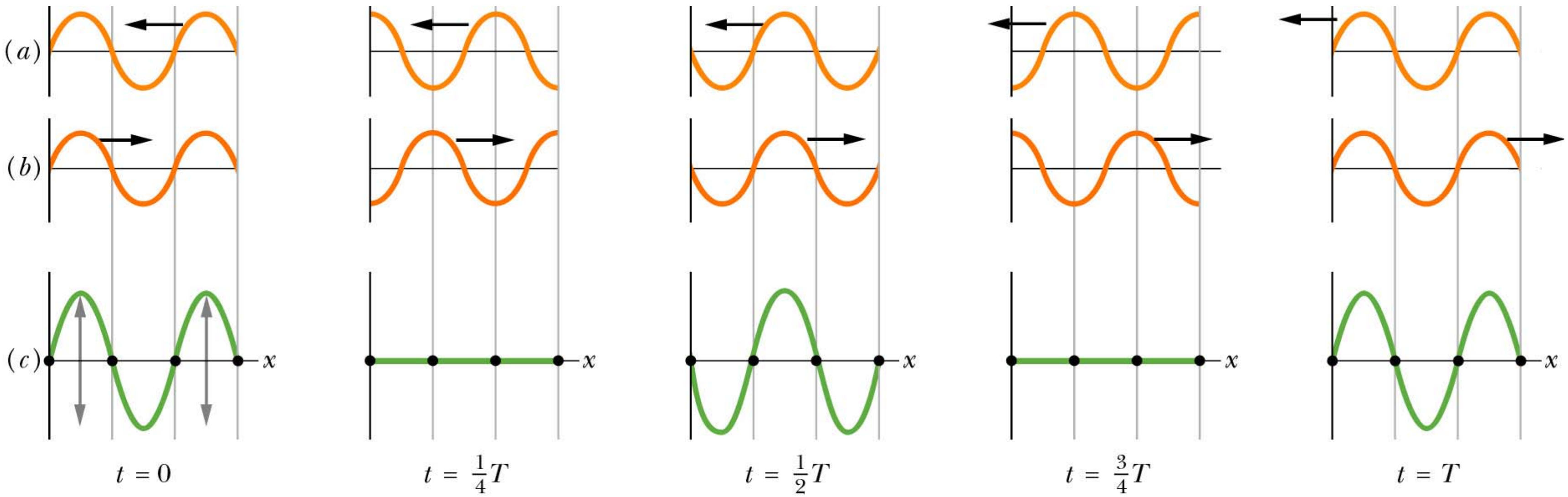
Phasors are useful in AC circuits and optics.

Standing waves

- The interference of two sinusoidal waves of the same frequency and amplitude, travel in opposite direction, produce a standing wave.

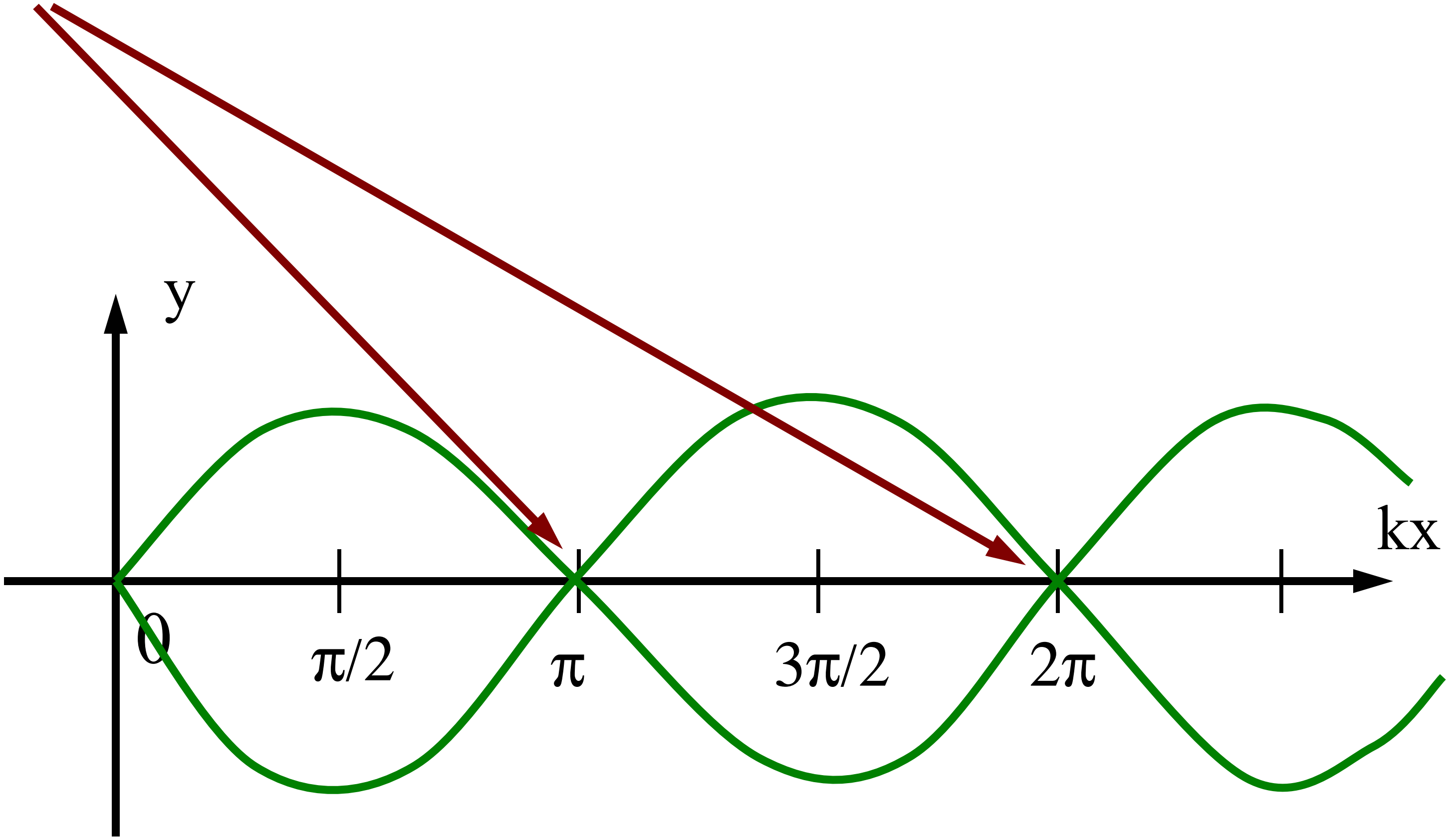
  $y_1(x, t) = y_m \sin(kx - \omega t)$, $\quad y_2(x, t) = y_m \sin(kx + \omega t)$

  resultant wave: $y'(x, t) = [2y_m \sin kx] \cos(\omega t)$



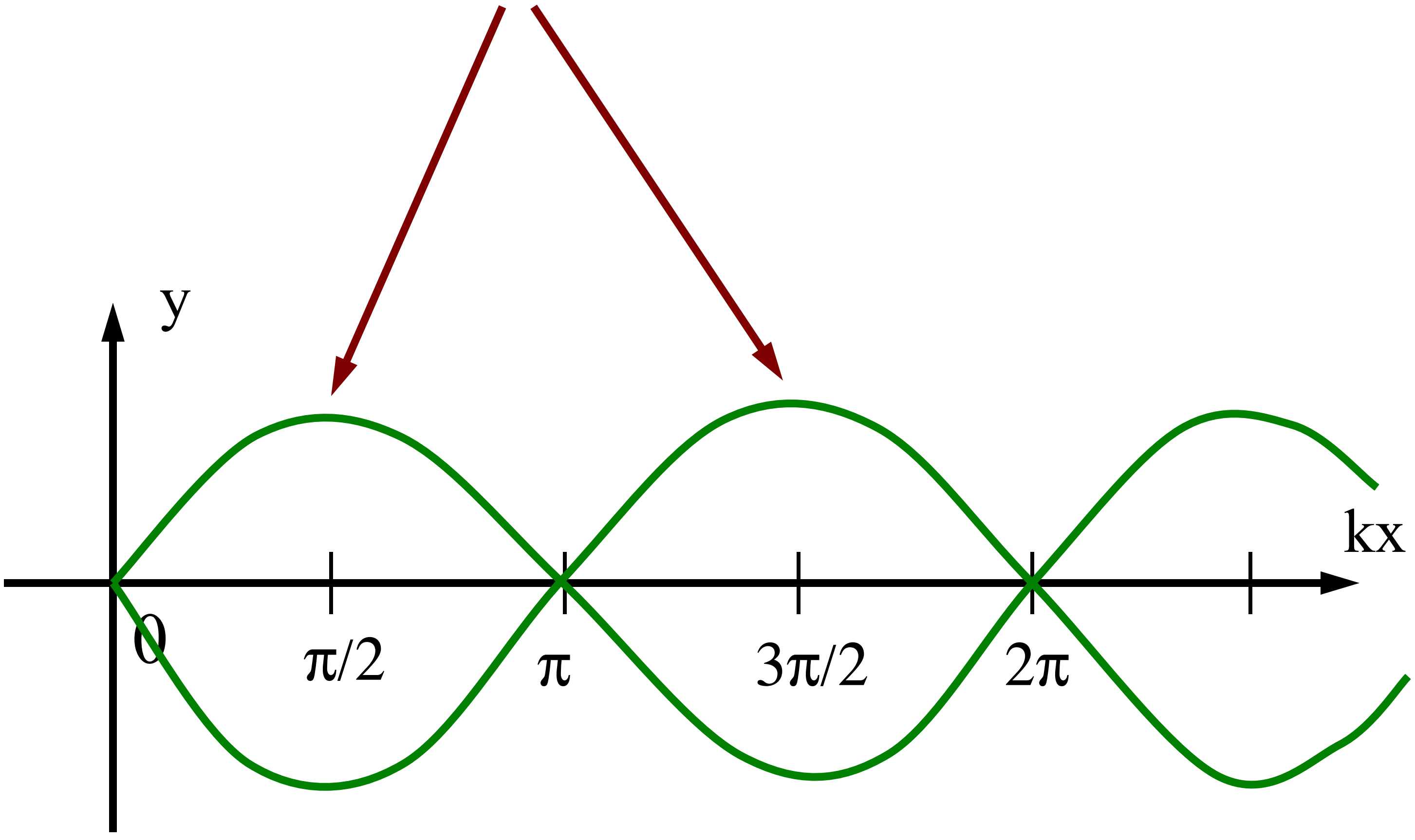
$$y'(x, t) = [2y_m \sin kx] \cos(\omega t)$$

If $kx = n\pi$ $(n = 0, 1, \ldots)$, we have $y' = 0$; these positions are called **nodes**. $x = n\pi/k = n\pi/(2\pi/\lambda) = n(\lambda/2)$



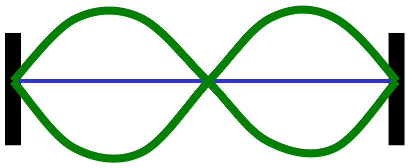
$$y'(x, t) = [2y_m \sin kx] \cos(\omega t)$$

If $kx = (n + ½)\pi$ $(n = 0, 1, \ldots)$, $y'_m = 2y_m$ (maximum);
these positions are called **antinodes**, $x = (n + ½)(\lambda/2)$



A Quiz

A standing wave exists on a wire that is stretched between two supports. How would you increase the frequency of the standing wave?



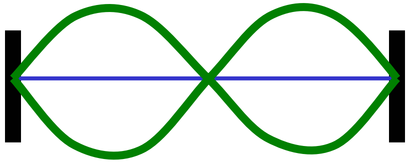
1) increase the tension
2) decrease the tension
3) increase the length of wire but with same tension
4) increase the mass density
5) none of the above

A Quiz

$$v = \sqrt{\frac{\tau}{\mu}} = f\lambda$$

increase tension increases v which increases f.

A standing wave exists on a wire that is stretched between two supports. How would you increase the frequency of the standing wave?



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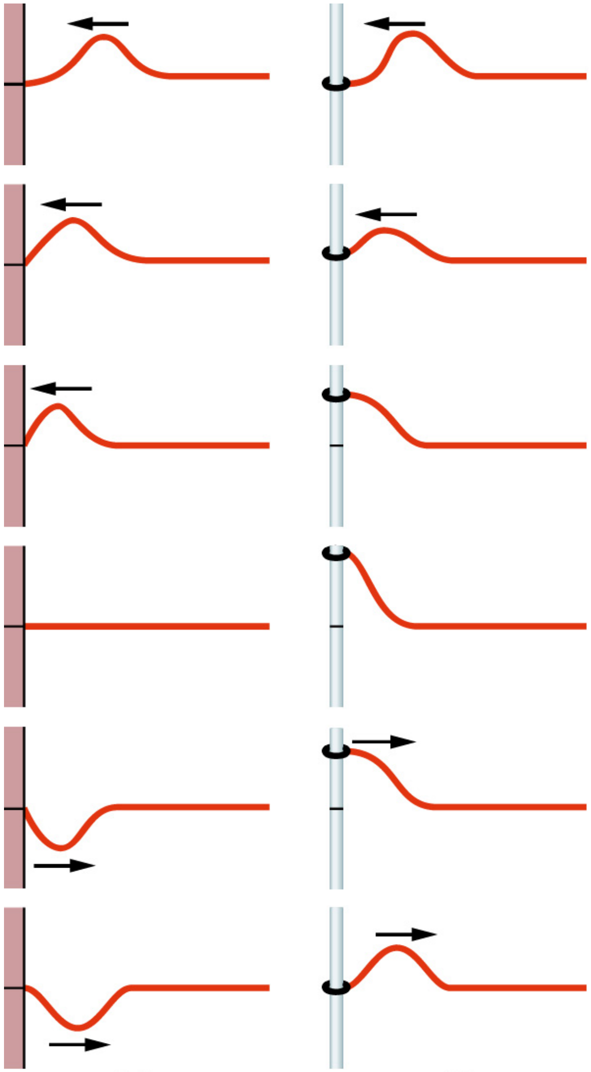
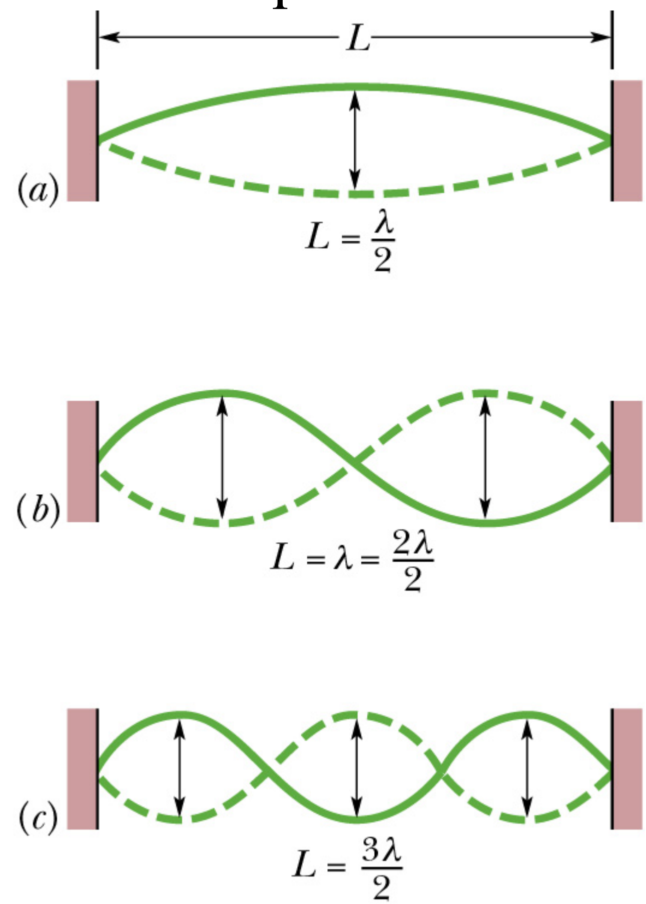
- Reflection at a boundary
  - In case (a), the string is fixed at the end. The reflected and incident pulses must have opposite signs. A node is generated at the end of the string.
  - In case (b), the string is loose at the end. The reflected and incident pulses reinforce each other.



(a) (b)

Standing wave and resonance

- For a string clamped at both end, at certain frequencies, the interference between the forward wave and the reflected wave produces a standing wave pattern. String is said to resonate at these certain frequencies, called resonance frequencies.

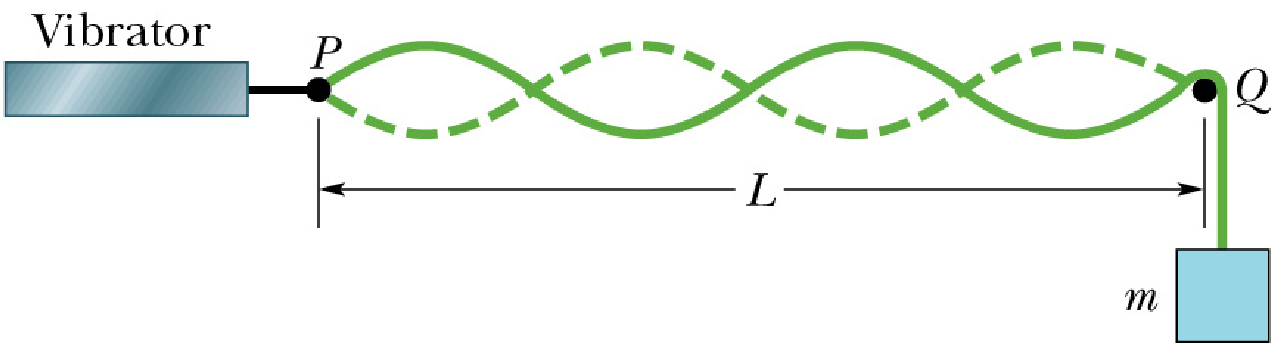


- $L = \lambda/2$, $f = v/\lambda = v/2L$
  1st harmonic, fundamental mode

- $L = 2(\lambda/2)$, $f = 2(v/2L)$
  2nd harmonic

- $f = n(v/2L)$, $n = 1, 2, 3\ldots$
  *n*th harmonic

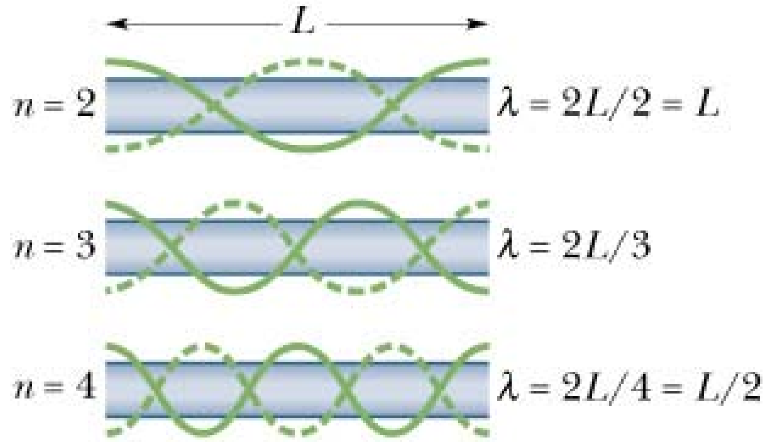
$L = 1.2$m, $\mu = 1.6$ g/m, $f = 120$ hz, points P and Q can be considered as nodes. What mass m allows the vibrator to set up the forth harmonic?



Pipes

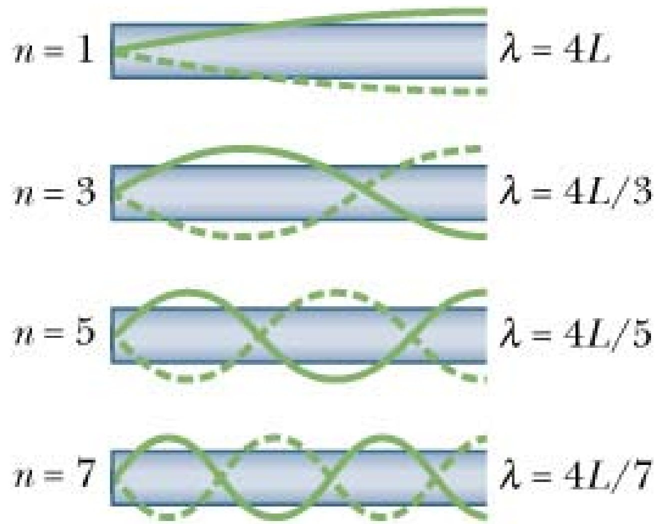
Open on both ends

Antinodes both ends.



(a)

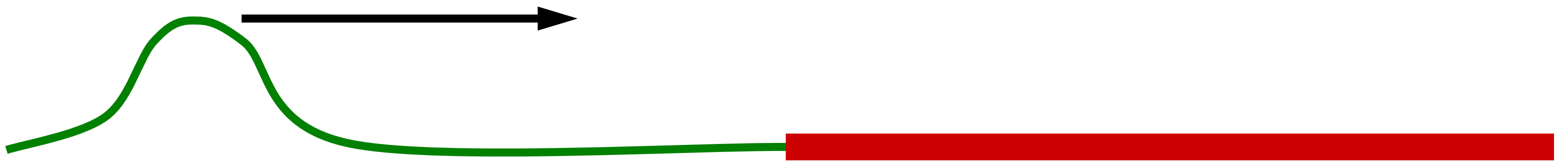
Open on one end,
closed on the other end



(b)

node on closed end,
antinode on open end.

A Quiz



A pulse is traveling down a wire and encounters a heavier wire. What happens to the reflected pulse?

1. The reflected wave changes phase
2. The reflected wave doesn't change phase
3. There is no reflected wave
4. none of the above

A Quiz

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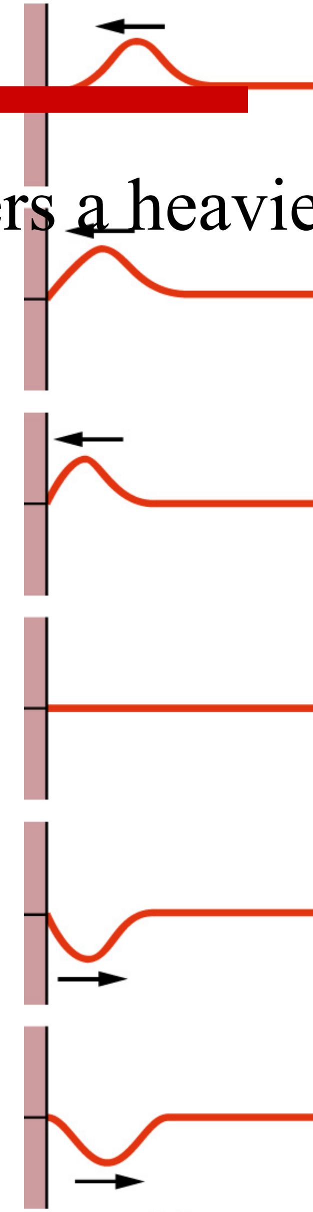

(a)