Examination III for PHYS 6220/7220, Fall 2010

1. A simple pendulum of mass m rests in static stable equilibrium in Toledo which has latitude θ. The magnitude of the acceleration due to gravity is g. The radius of the earth is R. The angular speed of rotation about its own axis is ω. Answer all questions only in terms of given quantities and assuming the pendulum remains in static stable equilibrium.

(a) Draw a figure showing the earth and pendulum and a convenient Cartesian frame at the center of the earth to answer the question in part (b). Label and define all important points on the figure clearly. (**1 point**)
(b) Find the effective force acting on the pendulum bob due to gravity and the rotation of the earth. Describe this force in terms of its components and unit vectors in the frame from part (a). (**2 points**)
(c) Find the angle the string of the pendulum makes with the local vertical direction at Toledo. (**1 point**)

2. A particle of mass m moves under gravity on a smooth surface which has an equation $z = x^2 + y^2 - xy$. The Z axis is taken along the vertical pointing upwards. The magnitude of the acceleration due to gravity is g.
(a) Write an expression for the potential energy of the particle purely as a function of x and y, $V = V(x, y)$. (**1 point**)
(b) Find the point (x_0, y_0) at which the potential is an extremum. (**1 point**)
(c) Compute the kinetic energy $T = T(x, y, \dot{x}, \dot{y})$. (**1 point**)
(d) Construct the appropriate Lagrangian for small oscillations about the point (x_0, y_0). (**1 point**)
(e) Find the eigen-frequencies for these oscillations. (**1 point**)
(e) Find the corresponding eigenvectors. (**2 points**)
(f) Find the most general solution. (**1 point**)
(g) Find the normal modes $Q_i = Q_i(\{q_i\})$, i = 1, … n where n are the total degrees of freedom in the problem. Depict the motion of these coordinates with arrows. (**2 points**)