Examination II for PHYS 6220/7220, Fall 2010

1. A Hamiltonian for a particle of mass m that moves in one dimension is given by

$H(x, p) = [(p^2)/m + m\omega^2x^2]/2$, where ω is a known real positive constant of appropriate dimensions.

(a) Contruct a canonical transformation Q = Q(x, p) and P = P(x, p) such that the Hamiltonian transforms to K(Q, P) where H(x, p) = K(Q, P) = $\imath\omega QP$ where $\imath \equiv \sqrt{-1}$ and Q and P are allowed to be complex. (**2 points**)

(b) Prove that your transformation is canonical. (**1 point**)

(c) Solve for Q(t) and P(t). (**1 point**)

(d) Using result in part (c) solve for x(t) and p(t). Comment on whether this answer is what is expected. (**1 point**)

2. Consider a general unit vector **n**. Any arbitrary vector **g** is rotated about the axis parallel to **n** by π radians to transform **g** to a new vector **f**. The matrix of this rotation is denoted as **A** so that **f** = **Ag**. Define two matrices $P_+ = (\mathbf{I} + \mathbf{A})/2$ and $P_- = (\mathbf{I} - \mathbf{A})/2$.

(a) Find the matrix $\mathbf{A}^2$ with the use of a figure depicting **g**, **f** and **n**. (**1 point**)

(b) From result in (a) we may express $P_+^2 = a\mathbf{I} + b\mathbf{A}$. Find the numerical constants a and b. (**1 point**)

(c) From result in (a) we may express $P_-^2 = e\mathbf{I} + f\mathbf{A}$. Find the numerical constants e and f. (**1 point**)

(d) From a figure of **n**, **g** and ($\mathbf{P}_+$)**g** and ($\mathbf{P}_-$)**g** deduce the physical interpretation of the two matrices $\mathbf{P}_+$ and $\mathbf{P}_-$. State it in words. Write expressions for ($\mathbf{P}_+$)**g** and ($\mathbf{P}_-$)**g** in terms of **n** and **g** alone. (**2 points**)

3. A particle of mass m, energy E and angular momentum ℓ exists in a central potential given by $V(r) = (k_1/r) + (k_2/r^2)$ where k_1 and k_2 are positive constants of appropriate dimensions. Provide all answers in given quantities only.

(a) Describe briefly the qualitative nature of the orbits in this potential. (**1 point**)

(b) Find the equation of the orbit r = r(θ). Express your answer in terms of constants b, c, and e given by

$b \equiv \frac{2}{k_1}\left(k_2 + \frac{l^2}{2m}\right), c \equiv \sqrt{1 + \frac{2mk_2}{l^2}}$ and $e \equiv \sqrt{1 + \frac{4E}{k_1^2}\left(k_2 + \frac{l^2}{2m}\right)}$. (**3 points**)

(c) Express time t as $t = \int f(q)dq$ where q is a generalized coordinate. Find the form of f(q). Do not solve the integral. State if it cannot be solved. If it can be solved point out which integral from our given table of integrals you would use. (**2 points**)

(d) Find the scattering angle from the result in part (b). (**2 points**)

(e) Now assume that the particle approaches the potential with speed v_0 from a far away point and its impact parameter is s. Describe how you would obtain the differential cross section of the particle from result in part (d). You need not perform this calculation but describe it clearly so that it may be performed if needed. (**2 points**)