# Examination I for PHYS 6220/7220, Fall 2010

1. A hemisphere of mass M and radius a rests with its flat surface on a frictionless horizontal plane as shown in Fig. 1. A mass m initially at rest at the top of the frictionless hemisphere loses its position of unstable equilibrium at time t = 0 and starts sliding on the surface of the hemisphere under the influence of gravity. Magnitude of the acceleration due to gravity is g. The X and Z axis are as shown. The radial vector to mass m from the center of the hemisphere makes an angle $\theta$ with the Z axis, at time t, as shown.

(a) Define clearly an appropriate set of generalized coordinates, in words. Use these to obtain the Lagrangian of the system. (**2 points**)

(b) Use a Lagrange multiplier $\lambda$ associated with the constraint that m moves over the hemisphere and write the Euler-Langrange equations of motion. (**2 points**)

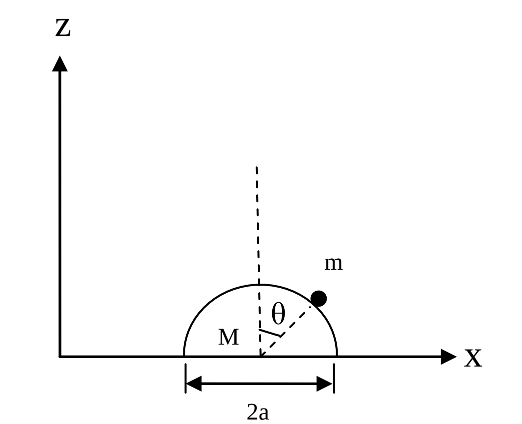

**Fig. 1**

(c) Write an expression for $\lambda$ purely as a function of only one generalized coordinate and its derivatives with time. (**2 points**)

(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)

(e) Eliminate the time derivatives in part (c) to express $\lambda$ purely in terms of one generalized coordinate. (**2 points**)

2. An ant is located at point $P_1$ with Cartesian coordinates ($x_1$, $y_1$, $z_1$) on a surface which is limited to the region z > 0. The surface has its defining equation as $z = s|(x^2 + y^2)^{1/2}|$, where s is a positive constant. The ant wants to remain on the surface and walk to a final destination point $P_2(x_2, y_2, z_2)$. Express all answers in given quantities only. Answer both parts for all possible points $P_1$ and $P_2$ on the surface.

(a) Find the exact curve that it should walk to complete its journey in the shortest distance. (**8 points**)

(b) Find the total distance the ant will travel along this curve. (**2 points**)