Examination III for PHYS 6220/7220, Fall 2010

1. A thin uniform circular disc of mass m, with radius of length b rotates at a constant angular frequency ω about an axis through the center of the disc tilted by an angle θ with respect to the normal to the disc. The body axes are denoted by xyz and the laboratory axis by x'y'z'. The common origin O of both systems is at the center of the disc. The angle θ remains constant at all times. The z axis is normal to the plate and the z' axis is the axis of rotation. The x and y axes are chosen such that the three axes x, z' and z are coplanar at all times.

(a) Compute the moment of inertia matrix of the disc in the body frame. Comment on your result. (**4 points**)

(b) Express the angular velocity $\boldsymbol{\omega}$ of rotation in the laboratory frame and the body frame. (**1 point**)

(c) Compute the angular momentum of the disc in the body frame at all times. Compute the torque on the disc in the body frame. (**2 points**)

(d) Compute the torque on the disc in the laboratory frame. (**1 point**)

(e) Compute the angular momentum of the disc in the laboratory frame. (**2 points**)

2. A particle of mass m and charge e moves in a potential $V(r) = [m\Omega^2 r^2]/2$, where r is the radius of the particle and Ω is a constant of appropriate dimensions. It is simultaneously subjected to a constant electric field **E** pointing along the positive X axis and a constant magnetic field **B** pointing along the Z direction. Express all answers in known quantities only.

(a) Write an expression for the vector potential **A** and the scalar potential ϕ associated with these electromagnetic fields if it is known that, the y component of **A**, $A_y = (xB)/2$. (**1 point**)

(b) Write the Lagrangian for this system. (**3 points**)

(c) Write the Euler Lagrange equations for all the generalized coordinates (**3 points**)

(d) One of the equations in (c) will yield the equation for a well-known problem. Identify this generalized coordinate and solve its equation to get its most general solution. Let this generalized coordinate be labeled q_n where there are n degrees of freedom in the problem. (**1 point**)

(e) For one of the other coordinates, a shift in origin will simplify the equation. Give an expression for the shift and rewrite its equation in this shifted coordinate. Let this coordinate be q_1. (**1 point**)

(f) For all generalized coordinates $\{q_k;\ k = 1, 2, \ldots n-1\}$, except the one in part (d), substitute an oscillatory solution of the form $q_k = C_k \exp(i\omega t)$ in the equations obtained in parts (c) and (e). Here, $i = (-1)^{1/2}$, C_k are arbitrary unknown constants and t stands for time. The unknown parameter ω is yet to be determined. (**1 point**)

(g) From the n-1 equations obtained after the substitution in part (f) state the condition which will give the unknown parameter ω that was introduced. Give a reason for the choice of your condition. Solve for ω using this condition. (**2 points**)