#### Chapter 6: Oscillations

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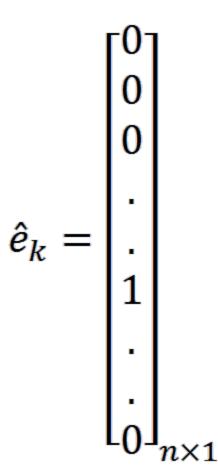
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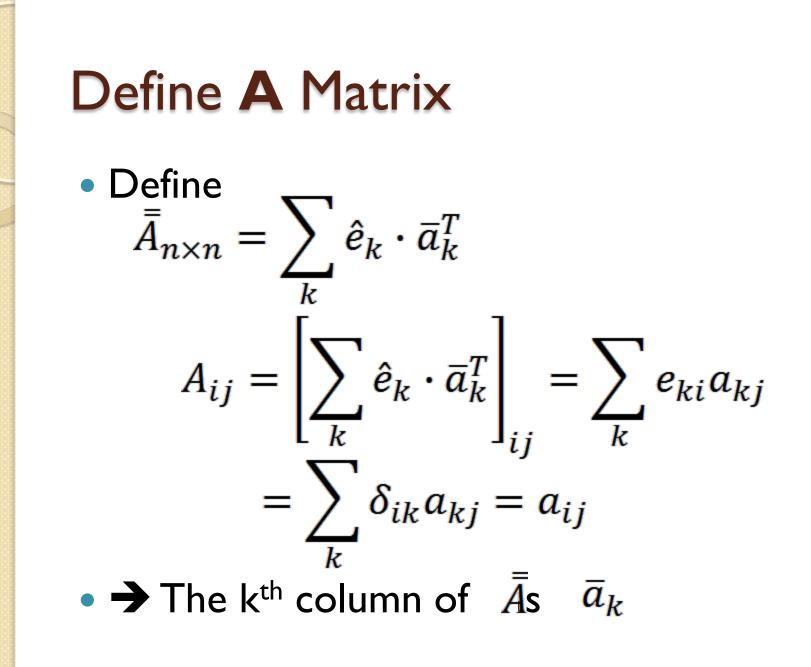
#### The Eigenvalue Problem

- Non-degenerate eigenvalues
- Let λ<sub>k</sub> be the n non-degenerate eigenvalues and <sup>6</sup>/<sub>ℓ</sub> the eigenvectors of <sup>¯</sup>/<sub>T</sub><sup>-1</sup><sup>¯</sup>/<sub>V</sub>
   <sup>¯</sup>/<sub>T</sub><sup>-1</sup><sup>¯</sup>/<sub>V</sub>
   <sup>¯</sup>/<sub>T</sub><sup>-1</sup><sup>¯</sup>/<sub>V</sub>
   <sup>¯</sup>/<sub>T</sub><sup>-1</sup><sup>¯</sup>/<sub>V</sub>
   <sup>¯</sup>/<sub>R</sub> = λ<sub>k</sub> ā<sub>k</sub>
- Where  $\bar{a}_{k} \equiv \begin{bmatrix} a_{k1} \\ a_{k2} \\ \vdots \\ \vdots \\ a_{k3} \end{bmatrix}$

### **Define Unit Vectors**

- Let  $\hat{e}_k$  be unit vectors such that
  - $e_{ki} = \delta_{ik}$
- In other words the vectors represented by an n x I matrix where all of its elements are zero except I at the k<sup>th</sup> row





## Use **A** in Eigenvalue Problem

- Original Form  $\overline{\overline{T}}^{-1}\overline{\overline{V}}\overline{a}_{k} = \lambda_{k}\overline{a}_{k}$  $\overline{\overline{V}}\overline{a}_{k} = \lambda_{k}\overline{\overline{T}}\overline{a}_{k}$
- Using  $\bar{\bar{A}}$ 
  - $\bar{\bar{V}}\bar{\bar{A}}=\bar{\bar{\lambda}}\bar{\bar{T}}\bar{\bar{A}}$
- Where  $\overline{\lambda}$  is a n x n matrix which is diagonal •  $\rightarrow$  $\lambda_{ij} = \lambda_i \delta_{ij}$



### **Useful Properties**

- Define the Hermitian conjugate by a dagger superscript  $\Box$  such that  $\overline{B}^{\dagger} \equiv \left(\overline{B}^{T}\right)^{*} \equiv \left(\overline{B}^{*}\right)^{T}$ where  $(\overline{B}^{*})_{ij} \equiv (B_{ij})^{*} \equiv B_{ij}^{*}$
- Where \* denotes complex conjugation  $(a + ib)^* = a - ib \text{ or } (re^{i\theta})^* = re^{-i\theta}$

where r is real •  $\overline{V}$  and  $\overline{\overline{T}}$  are real and  $\overline{V}^T = \overline{V} \& \overline{\overline{T}}^T = \overline{\overline{T}}$  $\rightarrow \overline{V}^\dagger = \overline{V} \& \overline{\overline{T}}^\dagger = \overline{\overline{T}}$ 

# **Applying to Eigen Equation**

- Starting with  $\overline{V}\overline{A} = \overline{\lambda}\overline{T}\overline{A}$
- Multiply both sides by  $\bar{\bar{A}}^{\dagger}$  $\bar{\bar{A}}^{\dagger}\bar{\bar{V}}\bar{\bar{A}} = \bar{\bar{A}}^{\dagger}\bar{\bar{\lambda}}\bar{\bar{T}}\bar{\bar{A}}$
- Hermitian conjugate of both sides  $\left(\bar{\bar{A}}^{\dagger}\bar{\bar{V}}\bar{\bar{A}}\right)^{\dagger} = \left(\bar{\bar{A}}^{\dagger}\bar{\bar{\lambda}}\bar{\bar{T}}\bar{\bar{A}}\right)^{\dagger}$
- Since  $\overline{V}^{\dagger} = \overline{V} \& \overline{\overline{T}}^{\dagger} = \overline{\overline{T}} \& \overline{\overline{A}}^{\dagger} = \overline{\overline{A}}^{\dagger}$  $\overline{\overline{A}}^{\dagger} \overline{V} \overline{\overline{A}} = \overline{\overline{A}}^{\dagger} \overline{\overline{T}} \overline{\overline{\lambda}}^{\dagger} \overline{\overline{A}}$

# Checking Properties of $\Box$

• From  $\bar{\bar{A}}^{\dagger}\bar{\bar{V}}\bar{\bar{A}} = \bar{\bar{A}}^{\dagger}\bar{\bar{T}}\bar{\bar{\lambda}}^{\dagger}\bar{\bar{A}}$ 

• Using gives  $\bar{\bar{A}}^{\dagger}\bar{\bar{\lambda}}\bar{\bar{T}}\bar{\bar{A}} = \bar{\bar{T}}\bar{\bar{\lambda}}^{\dagger}$ 

- And using  $\bar{\bar{\lambda}}\bar{\bar{T}} = \bar{\bar{T}}\bar{\bar{\lambda}}^{\dagger}$
- Which implies  $\overline{\lambda}$  is diagonal  $\overline{\lambda}\overline{\overline{T}} = \overline{\overline{T}}\overline{\lambda}^{\dagger} \to \overline{\lambda}\overline{\overline{T}} = \overline{\overline{T}}\overline{\overline{\lambda}} \to \overline{\overline{\lambda}} = \overline{\overline{\lambda}}^{\dagger}$

