1. Consider a simple pendulum at rest in Toledo. Let the mass of the bob be \( m \). Choose an appropriate coordinate system at the center of the earth. Answer all questions in this coordinate system. The radius of the earth is \( R \), and acceleration due to gravity is \( g \) and the rotational angular frequency of the earth is \( \omega \). Toledo is at a latitude \( \theta \).
(a) Find the total force on the bob due to gravity and the rotation of the earth. [3 points]
(b) Find the angles that the string of the pendulum makes with all three axes of your coordinate system. [3 points]
(c) What would be the answer to part (b) if we ignored the rotation of the earth. [1 point]

2. Two equal masses each of value \( m \) are connected to two identical springs with spring constant \( k \). As shown in the figure. One end of one of the springs is fixed at point A. The magnitude of the acceleration due to gravity is \( g \).
(a) Write the Lagrangian of the system by choosing an appropriate set of coordinate axes. Define and explain your coordinates clearly. [2 points]
(b) Find the eigenfrequencies for small oscillations of this system about its equilibrium configuration. [2 points]
(c) Find the corresponding eigenvectors. [2 points]
(d) Find the most general solution. [1 point]
(e) Show with arrows the relative phases of the particles in the normal modes. [1 point]