Third Examination for PHYS 6220/7220, Fall 2009

1. Consider a simple pendulum at rest in Toledo. Let the mass of the bob be m. Choose an appropriate coordinate system at the center of the earth. Answer all questions in this coordinate system. The radius of the earth is R, and acceleration due to gravity is g and the rotational angular frequency of the earth is ω. Toledo is at a latitude θ.
(a) Find the total force on the bob due to gravity and the rotation of the earth. [**3 points**]
(b) Find the angles that the string of the pendulum makes with all three axes of your coordinate system. [**3 points**]
(c) What would be the answer to part (b) if we ignored the rotation of the earth. [**1 point**]

2. Two equal masses each of value m are connected to two identical springs with spring constant κ. As shown in the figure. One end of one of the springs is fixed at point A. The magnitude of the acceleration due to gravity is g.
(a) Write the Lagrangian of the system by choosing an appropriate set of coordinate axes. Define and explain your coordinates clearly. [**2 points**]
(b) Find the eigenfrequencies for small oscillations of this system about its equilibrium configuration. [**2 points**]
(c) Find the corresponding eigenvectors. [**2 points**]
(d) Find the most general solution. [**1 point**]
(e) Show with arrows the relative phases of the particles in the normal modes. [**1 point**]

Fig. for problem 2