Examination I for PHYS 6220/7220, Fall 2009

1. An ant is located at the cylindrical coordinates (R, ϕ_1, z_1) on a tree trunk. We can approximate the trunk as a right circular cylinder of radius R. The ant notices a drop of honey at the point (R, ϕ_2, z_2) on the trunk. It wants to go to the drop by walking the shortest distance on the surface of the trunk.
(a) Using the calculus of variations find the equation of the path it should follow. Your final answer should only involve the given quantities. (**3 points**)
(b) Describe the special cases (i) $z_1 = z_2$ and (ii) $\phi_1 = \phi_2$ separately. (**2 points**)
(c) If your general solution in part (a) fails for either of the special cases in part (b) give reasons for the failure. (**1 point**)

2. A planar pendulum has a string of length b and mass m of the bob. It is known that at the initial time the mass m had only a horizontal velocity with magnitude $b\omega$ and the string was vertical. The magnitude of the acceleration due to gravity is g. All answers should only involve the given quantities.
(a) Use the method of Lagrange's undetermined multipliers to set up the Lagrangian for the system. Clearly define and state your coordinate system, the generalized coordinates and any constraints on the system. (**1 point**)
(b) Find the Lagrange's multiplier purely as a function of the generalized coordinates of the system. (**2 points**)
(c) Analyze the result in part (b) to obtain the minimum and maximum values that the multiplier may take. (**1 point**)
(d) From the result in part (c) and the physical interpretation of the multiplier determine the minimum or maximum value of ω for which the pendulum string will go slack. (**1 point**)
(e) From the result in part (b) determine the critical angle the string makes with the vertical at which point the string could go slack for a fixed value of ω? (**1 point**)

3. A charged particle of charge e is subject to an electro-magnetic field. The vector potential **A** that generates this field is given in Cartesian coordinates as (0, xB, 0) where B is a constant. The scalar potential Φ is zero.
(a) Write the Lagrangian for this system. (**1 point**)
(b) Write the Hamiltonian for the system. (**1 point**)
(c) Find the constants of motion that appear in the Hamiltonian. (**1 point**)
(d) Use the results in part (c) to reduce the dimensionality of the problem. (**1 point**)
(e) Now write Hamilton's equations for this reduced dimensionality problem. (**1 point**)
(f) Solve the equations in part (e). (**1 point**)
(g) Now solve for all three Cartesian coordinates. (**1 point**)
(h) Describe the nature of the trajectory found after completing part (g). (**1 point**)