The standard path for Lagrangian Problems *with undetermined Multipliers*

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Many students who have studied the theory behind solving mechanics problems, *with undetermined multipliers*, using the Lagrangian or Hamiltonian method find it very confusing at best. At worst they are simply unable to even get started on the problem. To guide such students we have designed an elaborate 22 step algorithm which will provide succor to get past conceptual hurdles. These are listed below.

1. **State the initial conditions and given quantities in writing, initially in sentences and once more expertise is attained in phrases or mere symbols.**
2. **Draw a sketch of the various parts of the system. Label as many special points as you can. Err on the side of too many labeled points but never on the side of too few.**
3. **Define an inertial frame in which the problem will be solved. State it in words. If you need to work in a non-inertial frame you should still always have an inertial frame defined.**
4. **Select the origin and Cartesian axes in this frame. Note all 3 Cartesian axes should always be defined even if the eventual problem may be solved in some other coordinates.**
5. **State the number of particles N and hence the 3N degrees of freedom (dof).**
6. **Write holonomic constraints for each identifiable constraint. Do not skip this step. If you get stuck here then struggle with it till you get a set of k holonomic constraints.**
7. **Note for a Lagrange's undetermined multiplier problem that $m \leq k$ of these constraints will have an associated multiplier. The**

problem will clearly state for which m of the k constraints one should use a multiplier (λ_i), where i = 1, 2, … m. If this is not given then pause, think and carefully figure the m constraints that have a multiplier that needs determination.

8. **With each holonomic constraint there will be a reduction in the dof in a normal problem. *This is not the case with multipliers. This is the most important tricky step.* There will now be (3N - k + m) dofs. This number is different from a normal problem because of multipliers. Note that you have an additional m degrees of freedom. Write these down in words or symbols for coordinates if possible.**
9. **Using the help of symmetries choose the generalized set of coordinates {q_i, I = 1, 2, … 3N – k + m}. *Note the extra m degrees are present in these problems.* Care should be taken in this choice so that the resulting equations in step 13 below will be simple. If you do not get a good choice now, work through step 13 and come back to this step to make a better choice of coordinates.**
10. **Write T, V, and then L'. Note L' = T - V. Note while writing T and V that you may first write them in a form which is convenient like in Cartesian or other coordinates. Then transform these expressions in generalized coordinates and velocities.**
11. **Now write *L = L' + $\lambda_i \sum_{i=1}^{m} f_i$, where $f_i = 0$, is the i^{th} holonomic constraint.***
12. **At this step pause and check various limits and dimensions of T and V. Correct any errors you may find.**
13. **Write the Euler- Lagrange Equations from the Lagrangian L. *Note you should have (3N - k + m) such equations.***
14. **Note now you may use the m holonomic constraints and their consequences in all the equations.**
15. **Solve these equations to the extent possible for the 3N – k coordinates and all the λ_i, i = 1, 2, … m. Often you will have to find the multipliers first as a function of the generalized coordinates**

and velocities. Then you may need to use significant mathematical manipulation to eliminate the velocities to get the multipliers purely as a function of the coordinates.

16. Plug in initial conditions if any special ones are given.

17. Take a pause after the solution to check limits and dimensions.

18. Check that your constraints are satisfied by your solution. ***Note that all k holonomic equations should now hold true.***

19. ***Analyze how different types of motion may arise with limiting values of*** λ_i, **i = 1, 2, … m.**

20. Search for constants of the motion if any.

21. Comment on your results, in terms of any new or old physics you may have observed in this problem. Any connections to other problems in mechanics or other branches of science.

22. Make notes on what new physics or math you have learned in the process of solving this problem for future reference and study.