Examination III for PHYS 6220/7220, Fall 2008

In this test vectors and matrices are denoted by bold face characters.

1. Two Cartesian coordinate systems S and S' share a common origin and Z axis. S' is always at rest with respect to S. The axes of system S' can be obtained by rotating system S about the Z axis by an angle θ. The rotation matrix that takes the frame S to S' is called **R**. Both frames are fixed onto a planar lamina of undetermined shape such that it lies in the XY plane of S. In the frame S the lamina has an angular velocity of rotation $\boldsymbol{\omega}$ and moment of inertia matrix $\mathbf{I_M}$. In the frame S' the lamina has an angular velocity of rotation $\boldsymbol{\omega}'$ and moment of inertia matrix $\mathbf{I_M}'$. One point on the lamina is held fixed at the origin of S.
(a) Express the kinetic energy T' of the lamina in terms of the primed quantities only. (**1 point**)
(b) Express the kinetic energy of the lamina T in terms of the unprimed quantities only. (**1 point**)
(c) What is the relationship of T to T'? (**1 point**)
(d) Write a relation between $\boldsymbol{\omega}'$ and $\boldsymbol{\omega}$. (**1 point**)
(e) Combine the results of (c) and (d) to obtain a relationship between $\mathbf{I_M}'$ and $\mathbf{I_M}$. (**1 point**)
(f) Three components of $\mathbf{I_M}$ are given by, $I_{M11} = a$, $I_{M22} = b$, and $I_{M12} = c$. Express all other components of $\mathbf{I_M}$ in terms of these three. (**1 point**)
(g) Construct $\mathbf{I_M}'$ in terms of a, b, c and the angle θ. (**3 points**)
(h) If S' is known to be the principal axes of the lamina then express θ in terms of a, b, and c? (**1 point**)

2. A rod of mass m uniform mass density and length ℓ is let to stand vertically on a horizontal floor. The rod is given an infinitesimal displacement so that it starts falling. The magnitude of the acceleration due to gravity is g.
(a) Assume that the point of contact to the floor is held fixed. Find the angular speed ω_a with which it hits the floor. (**2 points**)
(b) Now assume as an alternative to part (a) that the point of contact is not held fixed and the floor is frictionless. In this case find the angular speed ω_b with which it hits the floor. (**2 points**)

3. A graduate student who had just finished PHYS 6220 had studied small oscillations very well. In an electromagnetism course he encountered two circuits for which, using Kirchoff's laws, he arrived at the following equations for charges q_1 and q_2 on two capacitors.

$(L\,\ddot{q}_1) + (q_1 / C) + (M\,\ddot{q}_2) = 0$ and $(L\,\ddot{q}_2) + (q_2 / C) + (M\,\ddot{q}_1) = 0$, where L, C, and M are constants of appropriate dimensions. He knew that the two charges could vary independently in time. The problem was to solve for the most general solution to the time dependence of the two charges $q_1(t)$ and $q_2(t)$. Follow the steps that he took by answering the following.

(a) He first identified a kinetic energy and potential energy term treating the two charges as the generalized coordinates. Find these. (**2 points**)
(b) He then wrote the Lagrangian for the system. He solved for part(a) in a manner that the given equations were the Euler-Lagrange equations for his problem. Write the Lagrangian and confirm that the given equations can be derived from it naturally. (**1 point**)
(c) He then constructed the **T** and **V** matrices to make the problem appear as a small oscillations problem. Write these matrices. (**2 points**)
(d) Using the results of part(c) he solved for the eigen-frequencies of the problem. Find these. (**2 points**)
(e) Using the results of part(d) he solved for the eigen-vectors of the problem. Find these. (**2 points**)
(f) He then proceeded to write the most general solution for the original equations he was to solve. Obtain this solution. (**1 point**)