# Examination II for PHYS 6220/7220, Fall 2008

1. A system of two degrees of freedom is described by a Hamiltonian $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, where b and a are constants. Two functions are defined by $F_1 = (p_1-aq_1)/q_2$ and $F_2 = q_1q_2$.
(i) Calculate the partial derivatives of these two functions with time. (**1 point**)
(ii) Use the result in part (i) to calculate $[F_1, H]$. (**2 points**)
(iii) Use the result in part (i) to calculate $[F_2, H]$. (**1 point**)
(iv) From results in parts (i-iii) what can you say about these two functions? (**1 point**)

2. Consider two unit vectors $\mathbf{n_A} = (1,1,0)/\sqrt{2}$ and $\mathbf{n_B} = (0, 0, 1)$. A vector is first rotated by an angle of $\pi$ radians about an axis parallel to $\mathbf{n_A}$. After that the resulting new vector is reflected in a plane normal to $\mathbf{n_B}$.
(i) Find all elements of the matrix **A** corresponding to the rotation. (**1 point**)
(ii) Find all elements of the matrix **B** corresponding to the reflection. (**1 point**)
(iii) If the resulting total change of the vector, due to the successive rotation and reflection, is represented by a matrix **R** then find all its elements. (**1 point**)
(iv) Does **R** correspond to a pure rotation? Justify your answer. If **R** does indeed correspond to a pure rotation find the corresponding angle of rotation. If **R** does not correspond to a pure rotation then does **R** correspond to a pure reflection? Justify your answer. If so, what is the unit normal to the plane of reflection? (**2 points**).

3. A particle of mass m, energy E and magnitude of angular momentum $\ell$, moves in a central force potential $V(r) = (k_1/r) + (k_2/r^2)$, where $k_1$ and $k_2$ are positive constants.
(a) Derive the most general form of the equation of the orbit $r = r(\theta)$, using the integral form of the equation for the orbit. (**2 points**)
(b) Analyze the orbit equation from part (a) to obtain the limits on r. (**2 points**)
(c) From result in part (b) find the scattering angle for the unbound orbits. Express this in terms of the impact parameter s. (**2 points**)

4. A particle of mass m moves in a potential $V(r) = -V\exp[-(r/a)^2]$, where V and a are positive constants of appropriate dimensions. Find the critical value $\ell_c$, of its angular momentum about the center of force below or above which bounded orbits are possible. Explain your reasoning clearly. The final answer can only involve the given constants in the problem, m, V, and a. (**4 points**)