Final Examination for PHYS 6220/7220, Fall 2008

All final answers should be in terms of given quantities only.

1. Two points have cylindrical coordinates \((R, \phi_1, z_1)\) and \((R, \phi_2, z_2)\), since they both lie on a cylindrical surface of radius \(R\). Derive the equation of the curve that gives the shortest distance between these two points as traveled on the cylindrical surface. The curve has a specific geometrical name. State it. (5 points)

2. A particle of mass \(m\) and angular momentum \(\ell\) moves in a central force field and has an equation of the orbit given by \(r = a(1 + \cos(\theta))\), where \(a\) is a positive constant of appropriate dimensions.
   (a) Find the form of the potential \(V(r)\). (2 points)
   (b) A particle of mass \(M\) approaches this center of force from far away with initial speed \(V_0\). What is the critical value of the impact parameter \(b\) above which the particle fails to reach the origin? Ignore the gravitational influence of the first particle of mass \(m\). (3 points)

3. A carousel starts from rest and accelerates with constant angular acceleration \(\alpha\). A girl sitting on a chair a distance \(r_0\) from the axis of rotation holds a ball of mass \(m\). Calculate the magnitude and direction of the force she must exert to hold the ball a time \(T\) after the carousel begins to rotate. State your directions with respect to the radial vector from the center of rotation to the chair. The magnitude of the acceleration due to gravity is \(g\). (5 points)