Mechanics is the study of particles & systems of particles.

Classical - denotes pre-quantum mechanics. Includes special and general relativity.

$\bar{r}$ = position vector
$\bar{V}$ = ~~vec~~ velocity

$$\bar{V} \equiv \frac{d\bar{r}}{dt} \quad , \quad \bar{p} \equiv m\bar{V}$$

$$\bar{F} = \frac{d\bar{p}}{dt} = \dot{\bar{p}} \longrightarrow (1.3)$$

If m = constant then

$$\bar{F} = m\bar{a} \quad \text{where } \bar{a} \equiv \frac{d\bar{V}}{dt} = \frac{d^2\bar{r}}{dt^2}$$

$$\therefore \bar{a} \equiv \dot{\bar{V}} \equiv \ddot{\bar{r}}.$$

When (1.3) holds that frame of reference is an inertial system.

$$\bar{F} = \dot{\bar{p}}$$

$$\bar{r} \times \bar{F} = \bar{r} \times \dot{\bar{p}} = \bar{r} \times \frac{d\bar{v}}{dt} m$$

$$+ \bar{v} \times m\bar{v}$$

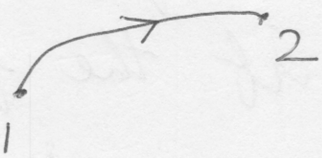
$$= \frac{d}{dt}(\bar{r} \times \bar{v} m) = \frac{d\bar{L}}{dt}$$

where $\bar{L} \equiv m\,\bar{r} \times \bar{v}$

$$\bar{N} \equiv \bar{r} \times \bar{F} = \frac{d\bar{L}}{dt} = \dot{\bar{L}}$$

If $\bar{N} = \bar{0}$ then $\bar{L}$ is conserved. Conservation of angular momentum. If $\bar{F} = \bar{0}$ then $\bar{p}$ is constant. Conservation of linear momentum.

$$W_{12} \equiv \int_1^2 \bar{F} \cdot \overline{ds}$$



$$= m \int_1^2 \left(\frac{d\bar{v}}{dt}\right) \cdot \left(\bar{v}\, dt\right) = \frac{m}{2} \int_1^2 \frac{d(\bar{v}^2)}{dt}\, dt$$

$$= \frac{m}{2}\left(\bar{v}_2^{\,2} - \bar{v}_1^{\,2}\right)$$

$$\therefore \; W_{12} = T_2 - T_1$$

A force is conservative if

$$\oint \bar{F} \cdot \overline{ds} = 0$$

By vector analysis then

$$\bar{F} = -\bar{\nabla} V(\bar{r})$$

$$\therefore \; W_{12} = -\int_1^2 \left[\bar{\nabla} V(\bar{r})\right] \cdot \overline{ds}$$

$$= -V_2(\bar{r}) + V_1(\bar{r}) = -V(\bar{r}_2) + V(\bar{r}_1)$$

$$= V_1 - V_2 = T_2 - T_1$$

$$\ni \; T_1 + V_1 = T_2 + V_2$$

⇒ Energy Conservation.

If the forces acting on a particle are conservative then the total energy defined as $T+V$ is conserved.

Mechanics of a system of particles

$$\left(\sum_j \bar{F}_{ji}\right) + F_i^e = \dot{\bar{p}}_i$$

$$\sum_i \dot{p}_i = \frac{d^2}{dt^2} \sum_i m_i r_i = \sum_i F_i^e + \sum_{\substack{i,j \\ i \neq j}} F_{ji}$$

Define $$\bar{R} = \frac{\sum_i m_i \bar{r}_i}{\sum_i m_i} = \frac{\sum_i m_i \bar{r}_i}{M}$$

$$\therefore \sum_i \dot{\bar{p}}_i = M \frac{d^2 \bar{R}}{dt^2} = \sum_i F_i^e$$

where we assumed $\bar{F}_{ij} = -\bar{F}_{ji}, \ \forall i,j$

⇒ Irrelevance of internal forces

$\bar{F}_{ij}$ for the center of mass (COM) motion:

$$\sum_i m_i \bar{v}_i = M \frac{d\bar{R}}{dt} = \bar{P}$$

If $\bar{F}^e = \bar{0} \Rightarrow \bar{P} = \text{constant}$

$\Rightarrow$ Conservation of total linear momentum.

$$\sum_i \bar{r}_i \times \dot{\bar{p}}_i = \sum_i \frac{d}{dt}\left(\bar{r}_i \times \bar{p}_i\right) = \frac{d\bar{L}}{dt} = \dot{\bar{L}}.$$

where $\bar{L} \equiv \sum_i \bar{r}_i \times \bar{p}_i$

$$\therefore \dot{\bar{L}} = \sum_i \bar{r}_i \times \bar{F}_i^e + \sum_{\substack{i,j \\ i\neq j}} \left(\bar{r}_i \times \bar{F}_{ji}\right)$$

Consider $\bar{r}_i \times \bar{F}_{ji} + \bar{r}_j \times \bar{F}_{ij}$

$$= \left(\bar{r}_i - \bar{r}_j\right) \cdot \bar{F}_{ji} \equiv \bar{r}_{ij} \times \bar{F}_{ji}$$

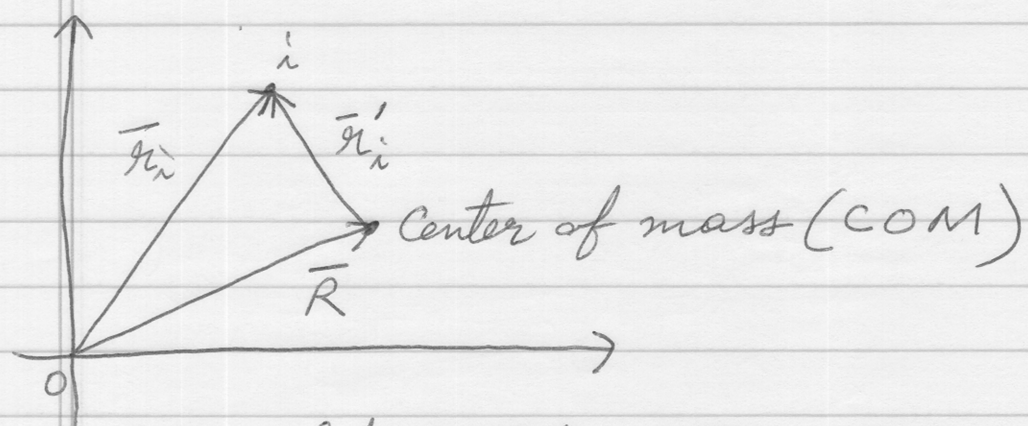
If $\bar{F}_{ji} \parallel \bar{r}_{ij}$ then

$$\bar{r}_{ij} \times \bar{F}_{ji} = \bar{0}$$

$$\therefore \dot{\bar{L}} = \sum_i \bar{r}_i \times \bar{F}_i^e = \bar{N}^e$$

$\Rightarrow$ Conservation of angular momentum.

Total angular momentum $\bar{L}$ = constant <u>iff</u> $\bar{N}^e = \bar{0}$.



Let $\bar{r}_i' = \bar{r}_i - \bar{R}$

$$\bar{V}_i' = \bar{V}_i - \bar{V}, \text{ where } \bar{V} \equiv \dot{\bar{R}}$$

$$\bar{V}_i' \equiv \dot{\bar{r}}_i'$$

$$\bar{L} \equiv \sum_i \bar{r}_i \times \bar{p}_i = \sum_i (\bar{r}_i' + \bar{R}) \times (\bar{V} + \bar{V}_i') m_i$$

$$= \sum_i \left[\bar{R} \times m_i \bar{V} + \bar{r}_i' \times m_i \bar{V}_i' \right]$$

$$+ \left(\sum_i m_i \bar{r}_i' \right) \times \bar{V} + \bar{R} \times \frac{d}{dt} \sum_i m_i \bar{r}_i'$$

$$\therefore \; \bar{L} \equiv \bar{R} \times M\bar{V} + \sum_i \bar{r}_i' \times \bar{p}_i'$$

$\ni$ Total angular momentum about an arbitrary origin O

= Angular momentum of COM about O,

+ angular momentum <u>about</u> COM.

Work done by a force.

$$W_{12}^S = \sum_i \int_1^2 \bar{F}_i \cdot d\bar{s}_i = \sum_i \int_1^2 \left(\bar{F}_i^e + \bar{F}_{ji}\right) \cdot d\bar{s}_i \longrightarrow 1.29$$

$$= \sum_i \int_1^2 m_i \bar{v}_i \cdot \dot{\bar{v}}_i \, dt = \sum_i \int_1^2 d\left(\frac{1}{2} m \bar{v}_i^2\right)$$

$$= T_2^S - T_1^S$$

where $T^S \equiv \sum_i \frac{1}{2} m_i \bar{v}_i^2$

$$= \sum_i \frac{1}{2} m_i \left(\bar{v}_i' + \bar{V}\right)^2 = \frac{M\bar{V}^2}{2} + \frac{1}{2} \sum_i m_i \bar{v}_i'^2$$

because $\sum_i m_i \bar{v}_i' \cdot \bar{V}$

$$= \bar{V} \cdot \sum_i \cancel{m_i} \frac{d}{dt}\left(\bar{r}_i' m_i\right) = \bar{V} \cdot \frac{d}{dt}\left(\sum_i m_i \bar{r}_i'\right)$$

$$\sum_i m_i \bar{r}_i' = \sum_i m_i \left(\bar{r}_i - \bar{R}\right)$$

$$= M\bar{R} - M\bar{R} = \bar{0}$$

$$\therefore T^S = \frac{1}{2} M \bar{V}^2 + \frac{1}{2} \sum_i m_i \bar{v}_i'^2$$

$\Rightarrow$ Kinetic ~~ener~~ energy of a system of particles T^S = Kinetic energy of its COM + Kinetic energy about COM.

Consider conservative form for $\bar{F}_i^e$

$$\Rightarrow \bar{F}_i^e = -\bar{\nabla}_i V_i$$

$$\therefore \sum_i \int_1^2 \bar{F}_i^e \cdot d\bar{S}_i = -\int_1^2 \sum_i \left(\bar{\nabla}_i V_i\right) \cdot d\bar{S}_i = \sum_i V_i \Big|_2^1$$

If $\bar{F}_{ij}$ are conservative then

$$\bar{F}_{ji} = -\bar{\nabla}_i V_{ij}$$

Strong form of Newton's 3^{RD} law

$$\Rightarrow \quad \bar{F}_{ji} = -\bar{\nabla}_i V_{ij} = \bar{\nabla}_j V_{ij} = -\bar{F}_{ij}$$

$$\Rightarrow \quad V_{ij}(\bar{r}_i, \bar{r}_j) = V_{ij}(|\bar{r}_i - \bar{r}_j|)$$

$$\bar{\nabla} V_{ij}(|\bar{r}|) = \bar{r} f \quad , f \text{ is a scalar.}$$

Consider now pairs of forces $\bar{F}_{ij}$ and $\bar{F}_{ji}$

$$\therefore \int_1^2 \left[\bar{F}_{ji} \cdot \bar{ds}_i + \bar{F}_{ij} \cdot \bar{ds}_j\right]$$

$$= \int_1^2 \left[-(\bar{\nabla}_i V_{ij}) \cdot \bar{ds}_i + (-\bar{\nabla}_j V_{ij}) \cdot \bar{ds}_j\right]$$

$$= \int_1^2 (\bar{\nabla}_i V_{ij}) \cdot (\bar{ds}_j - \bar{ds}_i)$$

$$\bar{ds}_j - \bar{ds}_i = -\bar{dr}_{ij} \equiv -(\bar{dr}_i - \bar{dr}_j)$$

Also $\quad \bar{\nabla}_i V_{ij} = \bar{\nabla}_{ij} V_{ij}$

$$\therefore \sum_{\substack{i,j \\ i\neq j}} \int_1^2 \bar{F}_{ji} \cdot \bar{ds}_i$$

$$= -\frac{1}{2} \sum_{\substack{i,j \\ i\neq j}} \int_1^2 (\bar{\nabla}_{ij} V_{ij}) \cdot \bar{dr}_{ij}$$

$$= -\frac{1}{2} \sum_{\substack{i,j \\ i\neq j}} V_{ij} \Big|_1^2$$

Define

$$V^S = \sum_i V_i + \frac{1}{2} \sum_{i,j}' V_{ij}$$

$$\therefore W_{12}^S = T_2^S - T_1^S$$

Also $$W_{12}^S = \sum_i V_i \Big|_2^1 + \frac{1}{2} \sum_{i,j}' V_{ij} \Big|_2^1$$

$$\therefore W_{12}^S = V_1^S - V_2^S$$

$$\Rightarrow T_2^S - T_1^S = V_1^S - V_2^S$$

$$V_1^S + T_1^S = V_2^S + T_2^S$$

$\Rightarrow$ Conservation of Energy for a system of particles