Examination II for PHYS 4310/5310, Spring 2006

1. The Hamiltonian for a particle in a one dimensional potential V(x) is given by $\hat{\mathbf{H}} \equiv \frac{\hat{\mathbf{P}}^2}{2m} + V(x)$. Your answers to both parts should only be expressed in terms of the two operators x and $\hat{\mathbf{P}}$ or their functions and constants such as m, and $\hbar$.

(a) Calculate the commutator $[x, \hat{\mathbf{H}}]$. (2 points)

(b) Write an expression for the minimum value of the product of uncertainties $\sigma_x \sigma_H$? (2 points)

2. An electron in the hydrogen atom has a wavefunction $\Psi(\vec{\mathbf{r}}, t) = (\Psi_{7,1,0} + \Psi_{3,0,0})/\sqrt{2}$. Find $\langle \sin(\theta) \rangle$, where θ is the polar angle of the position vector $\vec{\mathbf{r}}$ of the electron. (4 points)

3. A particle of mass m lies in the state $\Psi_n(x)$, which is the n$^{\text{th}}$ energy eigen-state of a one dimensional harmonic oscillator with eigen energy E_n, where n is a non-negative integer. The classical vibrational frequency of the particle is ω. Express all answers in terms of $\hbar$, ω, m, and n. Operators x and $\hat{\mathbf{P}}$ may also be used.

(a) Write an expression for the Hamiltonian operator $\hat{\mathbf{H}}$ of the system. (1 point)

(b) Calculate the expectation value of x^3. (1 point)

(c) Calculate the expectation value of the kinetic energy of the particle. (3 points)

(d) Calculate the expectation value of the operator x^2. (2 points)

4. A particle lies in a potential $V(\vec{\mathbf{r}}) = V(r)$, where $\vec{\mathbf{r}}$ is the position vector of the particle. In spherical polar coordinates $\vec{\mathbf{r}} = (r, \theta, \phi)$. One or more of the following options can be used to complete the statement below. For each option state whether the statement formed will be true or false. E stands for the energy of the particle, and L_x, L_y, and L_z for the components of its orbital angular momentum. L^2 is the square of the orbital angular momentum.

(3 points)

Statement: We can simultaneosly measure

(a) L_x and L^2 but not E.

(b) L_y and L^2 but not L_x.

(c) L_z and L_x but not L^2.

(d) L_y and E.

(e) L_z, L^2, and E.

(f) L_x, L_y, and L_z.

5. An electron in a hydrogen atom was measured to have a total energy $E = E_1/49$ where E_1 is its ground state energy. The z component of its orbital angular momentum was measured to be $-5\hbar$.

(a) Write a general expression for its wavefunction, in terms of the energy eigen-functions $\Psi_{n,\ell,m}$ of the problem. Ignore the spin part completely. Use as many arbitrary constants as you need but make sure the total wavefunction is normalized. (1 point)

(b) Calculate $\langle \hat{\mathbf{L}}^2 \rangle$ in terms of the arbitrary constants. (1 point)

Some relevant and irrelevant formulae are listed below.

$\int_0^\pi \sin(x)\cos^2(x)dx = \int_{-1}^1 x^2 dx$
