1. The Hamiltonian for a particle in a one dimensional potential $V(x)$ is given by $\hat{H} = \frac{\hat{P}^2}{2m} + V(x)$. Your answers to both parts should only be expressed in terms of the two operators $x$ and $\hat{P}$ or their functions and constants such as $m$, and $\hbar$.

(a) Calculate the commutator $[x, \hat{H}]$. (2 points)

(b) Write an expression for the minimum value of the product of uncertainties $\sigma_x \sigma_H$. (2 points)

2. An electron in the hydrogen atom has a wavefunction $\Psi(\mathbf{r}, t) = (\Psi_{1,1,0} + \Psi_{3,0,0})/\sqrt{2}$. Find $\langle \sin(\theta) \rangle$, where $\theta$ is the polar angle of the position vector $\mathbf{r}$ of the electron. (4 points)

3. A particle of mass $m$ lies in the state $\Psi_n(x)$, which is the $n^{th}$ energy eigen-state of a one dimensional harmonic oscillator with eigen energy $E_n$, where $n$ is a non-negative integer. The classical vibrational frequency of the particle is $\omega$. Express all answers in terms of $\hbar$, $\omega$, $m$, and $n$. Operators $x$ and $\hat{P}$ may also be used.

(a) Write an expression for the Hamiltonian operator $\hat{H}$ of the system. (1 point)

(b) Calculate the expectation value of $x^3$. (1 point)

(c) Calculate the expectation value of the kinetic energy of the particle. (3 points)

(d) Calculate the expectation value of the operator $x^2$. (2 points)

4. A particle lies in a potential $V(\mathbf{r}) = V(r)$, where $\mathbf{r}$ is the position vector of the particle. In spherical polar coordinates $\mathbf{r} = (r, \theta, \phi)$. One or more of the following options can be used to complete the statement below. For each option state whether the statement formed will be true or false. $E$ stands for the energy of the particle, and $L_x$, $L_y$, and $L_z$ for the components of its orbital angular momentum. $L^2$ is the square of the orbital angular momentum.

Statement: We can simultaneously measure

(a) $L_x$ and $L^2$ but not $E$.

(b) $L_y$ and $L^2$ but not $L_x$. 

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(c) $L_z$ and $L_x$ but not $L^2$.
(d) $L_y$ and E.
(e) $L_z$, $L^2$, and E.
(f) $L_x$, $L_y$, and $L_z$.

5. An electron in a hydrogen atom was measured to have a total energy $E = E_1/49$ where $E_1$ is its ground state energy. The $z$ component of its orbital angular momentum was measured to be $-5\hbar$.

(a) Write a general expression for its wavefunction, in terms of the energy eigen-functions $\Psi_{n,l,m}$ of the problem. Ignore the spin part completely. Use as many arbitrary constants as you need but make sure the total wavefunction is normalized. (1 point)
(b) Calculate $\langle \hat{L}^2 \rangle$ in terms of the arbitrary constants. (1 point)

Some relevant and irrelevant formulae are listed below.

$$\int_0^\pi \sin(x) \cos^2(x) dx = \int_{-1}^1 x^2 dx$$