Examination I for PHYS 4310/5310, Spring 2006

1. A free particle of mass m, is in an initial state $\Psi(x,0)$, which is an eigenstate of the position operator $\hat{\mathrm{X}}$, with eigenvalue b so that $\hat{\mathrm{X}}\Psi(x,0) = b\Psi(x,0)$.

(a) Write an expression for $\Psi(x,0)$. (1 point)

(b) This state may be expandend at all future time ($t > 0$), in terms of energy eigenfunctions in the form of the following integral.

$$\Psi(x,t) = \int_{-\infty}^{\infty} c(p)[\exp(ipx/\hbar)][\exp(if(p,t))]dp. \tag{1}$$

(b) Write an expression for the function f(p,t), involving p, t and other known constants of the problem? (1 point)

(c) Write the above equation at t = 0. Then multiply the left hand side by an appropriate function and integrate over all x to find c(p)? (2 points)

2. A particle has the normalized wavefunction $\Psi(x,0) = A\exp(-kx)$, when x > 0 and is zero everwhere else. k is a positive constant of appropriate dimensions.

(a) Find A in terms of k. (1 point)

(b) Find $\langle x \rangle$. (1 point)

(c) Find $\langle p \rangle$. (1 point)

(d) Find $\langle p^2 \rangle$. (1 point)

3. A particle of mass m lies in an infinite square well potential, with walls at $x = 0$ and $x = a$. It has an initial wavefunction $\Psi(x,0) = A(3\Psi_1(x) + 4\Psi_3(x))$. $\Psi_n(x)$, $n = 1, 2...$, are the energy eigen functions. Work out all parts only at time t = 0.

(a) Find A? (1 point)

(b) If a measurement is conducted for the particle energy what values may be obtained? Express your answer in terms of m, $\hbar$, and a. (1 point)

(c) If the energy is measured twice in succession, derive the probability of finding an energy $(3\pi\hbar)^2/(2ma^2)$, on the second measurement. Assume that you do not know the outcome of the first measurement. (2 points)

(d) After this second measurement where an energy $(3\pi\hbar)^2/(2ma^2)$ was obtained the position of the particle x was measured. What would be the probability of getting a value in the range $x_0 \pm (dx/2)$. (1 point)

4. A particle lies in the potential V(x) given by

$$V(x) = V_1, \quad \forall x < 0,$$
$$= 0, \quad \forall 0 \le x \le a$$
$$= V_2, \quad \forall x > a$$

V_1, V_2, and a are postive constants of appropriate dimensions with $V_1 < V_2$.

For this problem the time independent Schrödinger equation (TISE) takes the form,

$$\frac{d^2\Psi_E(x)}{dx^2} = C\Psi_E(x). \tag{2}$$

where E is the energy. The constant C will have different values in the three regions. In what follows restrict yourself to the case $0 < E < V_1$.

(a) Find C $\forall x < 0$. Then solve TISE for the most general form of $\Psi_E(x)$. Label your integration constants as A and B. (2 points)

(b) Find C $\forall 0 \le x \le a$. Then solve TISE for the most general form of $\Psi_E(x)$. Label your integration constants as C and D. (2 points)

(c) Find C $\forall x > a$. Then solve TISE for the most general form of $\Psi_E(x)$. Label your integration constants as F and G. (2 points)

(d) Identify any unphysical solutions if they exist in the general solution. Give a reason to say why they are unphysical. Which integration constants will be zero for the solutions to become physically meaningful? (1 points)

Some relevant and irrelevant formulae:

$\delta(k - k') = (2\pi\hbar)^{-1} \int_{-\infty}^{\infty} \exp\left(\frac{i(k-k')x}{\hbar}\right) dx.$

$\int_0^\infty x^n \exp(-ax)dx = (n!)/a^{1+n}, \forall n = 0, 1, 2, \ldots$