Final Examination for PHYS 4310/5310, Spring 2006

1. Two identical particles occupy exactly the same spin states. Therefore the symmetry of the spatial part of the state alone determines the symmetry of the entire state of the two particles. The spatial parts are made of two orthonormal single particle states $\Psi_a(x)$ and $\Psi_b(x)$, where the subscripts a and b denote the two different states. Define $\langle x\rangle_a \equiv \int_{-\infty}^{\infty} |\Psi_a(x)|^2 x dx$ and $\langle x\rangle_b \equiv \int_{-\infty}^{\infty} |\Psi_b(x)|^2 x dx$. In what follows express all wavefunctions in terms of $\Psi_a(x)$ and $\Psi_b(x)$ with appropriate subscripts 1 or 2 on x. Express all expectation values in terms of $\langle x\rangle_a$ or $\langle x\rangle_b$.

(a) Write an expression for the spatial part of the wavefunction $\Psi_F(x_1, x_2)$ if the two particles are fermions. Find $\langle (x_2 - x_1)\rangle_F$. (3 points)

(b) Write an expression for the spatial part of the wavefunction $\Psi_B(x_1, x_2)$ if the two particles are bosons. Find $\langle (x_2 - x_1)\rangle_B$. (3 points)

(c) Write an expression for the spatial part of the wavefunction $\Psi_D(x_1, x_2)$ if the two particles are distinguishable. Find $\langle (x_2 - x_1)\rangle_D$. (3 points)

2. Two particles each of mass m, lie in an infinite square well potential, so that $V(x) = 0, \forall 0 < x < a$ and $V(x) = \infty$, otherwise. Define $K \equiv [(\pi\hbar)^2]/(2ma^2)$. Express all energies in your answers in units of K. Let $\Psi_n(x)$ denote the one particle energy eigenstates for this potential, where n is a positive integer. All questions below refer to the two particle system.

(a) If the two particles are distinguishable write the energy of the ground state energy and that of the first excited state. (1 point)

(b) If the two particles are bosons write the energy of the ground state and that of the first excited state. (1 point)

(c) If the two particles are fermions write the energy of the ground state and that of the first excited state. (1 point)

(d) Write the energy eigen-functions corresponding to the energies listed in parts (a), (b), and (c) in terms of the $\Psi_n(x)$ with appropriate integer values of n, and appropriate subscrpt 1 or 2 for x. (3 points)

3. Assume that an atom contributes q free electrons to a continuous periodic one dimensional

solid. The toal number of atoms in the solid is N.

(a) How many electronic states are occupied in filling one allowed energy band? (1 point)

(b) If q = 3, would this solid be a conductor or insulator? (1 point)

4. A particle in one dimension, is in the ground state of a potential $V(x) = -\alpha\delta(x)$, where $\alpha > 0$ and $\delta(x)$ is the Dirac delta function.

(a) Write (you need not derive) the ground state wavefunction $\Psi(x,t)$ of the particle. (1 point)

(b) Find the expectation value of the Hamiltonian $\langle \hat{\mathbf{H}} \rangle$. and hence the expectation value of the kinetic energy $\langle \hat{\mathbf{T}} \rangle$ in this state. (3 points)

5. A particle has a normalized wavefunction given by $\Psi(r,\theta,\phi,0) = \{R(r)[Y_4^{-3}(\theta,\phi) + Y_3^2(\theta,\phi)]\}/\sqrt{2}$. It is given that $\int_0^\infty |rR(r)|^2 dr = 1$. Two operators are defined as $\hat{\mathbf{A}}_\pm \equiv \hat{\mathbf{L}}_x \pm \hat{\mathbf{L}}_y$, in terms of the x and y components of the angular momemtum operator $\hat{\vec{\mathbf{L}}}$.

(a) Express the commutator $[\hat{\mathbf{A}}_+, \hat{\mathbf{A}}_-]$ in terms of $[\hat{\mathbf{L}}_x, \hat{\mathbf{L}}_y]$. (1 point)

(b) Express the result in (a) as $c_x\hat{\mathbf{L}}_x + c_y\hat{\mathbf{L}}_y + c_z\hat{\mathbf{L}}_z$. Find the coefficients c_x, c_y, and c_z (1 point)

(c) Find a lower limit on the product of uncertainties $\sigma_{A_+}\sigma_{A_-}$ in the given state. (2 points)

6. The electronic configurations of three different atoms are denoted by the following symbolic codes. State which of these are possible and which are not. Give reasons for your answers?

(a) $^3P_0$, (b) $^5D_3$, and (c) $^3F_1$. (3 points)

7. A state of three identical particles each of half-integer spin, is made of the spatial part and the spin part. The spatial part is to be made of either $\Psi_S(1,2,3)$ or $\Psi_A(1,2,3)$. The spin part is made of either $\chi_S(1,2,3)$ or $\chi_A(1,2,3)$. The subscripts S and A stand for symmetric and anti-symmetric respectively. Construct all possible total states of the particles from these spatial and spin parts. (2 points)