1. Two particles have total spin quantum numbers $s_1 = 5/2$ and $s_2 = 1$, respectively. The two spins may be added to obtain new spin states which are simultaneous eigen-states of the total spin $\hat{S} \equiv \hat{S}_1 + \hat{S}_2$ and $\hat{S}_z$.

(a) List the possible values of the total spin quantum numbers $s$, of states obtained by adding the two spins. (1 point)

(b) List all the possible values of the quantum numbers $m$, corresponding to the $z$ component of the new composite spins. (1 point)

(c) List each of the possible spin states in the form $|s, m\rangle$. (3 points)

2. An electron in the hydrogen atom has a wavefunction $\Psi(\vec{r}, t) = (\Psi_{1,0,0} + \Psi_{5,1,0})/\sqrt{2}$. Find $\langle \sin(\theta) \rangle$, where $\theta$ is the polar angle of the position vector $\vec{r}$ of the electron. (4 points)

3. A particle lies in the state $|n\rangle$, which is the $n$th energy eigen-state of a one dimensional harmonic oscillator with eigen energy $E_n$, where $n$ is a non-negative integer. The classical vibrational frequency of the particle is $\omega$.

(a) Write the quantum Hamiltonian $\hat{H}$ for the system in the position basis in terms of the mass $m$ of the particle, $\omega$, and $\hbar$. You need not derive it. (2 points)

(b) What is the expectation value of $\hat{H}$, in terms of $\hbar$, $\omega$ and $n$. (2 points)

(c) Calculate the expectation value of the kinetic energy of the particle. (3 points)

4. A particle lies in a potential $V(\vec{r}) = V(r)$, where $\vec{r}$ is the position vector of the particle. In spherical polar coordinates $\vec{r} = (r, \theta, \phi)$. One or more of the following options can be used to complete the statement below. List all the options which will form a true statement. (3 points)

Statement: We can simultaneously measure any of the physical properites of the particle corresponding to

(a) $L_z$ and $L^2$ but not $E$.

(b) $L_y$ and $L^2$ but not $L_z$. 

1
(c) \( L_z \) and \( L_x \) but not \( L^2 \).

(d) \( L_x \) and \( E \).

(e) \( L_z, L^2, \) and \( E \).

5. An electron in a hydrogen atom was measured to have a total energy \( E = E_1/25 \) where \( E_1 \) is its ground state energy. The \( z \) component of its orbital angular momentum was measured to be \(-3\hbar\). Write a general expression for its wavefunction, in terms of the energy eigen-functions \( \Psi_{n,l,m} \) of the problem. (1 point)

Some relevant and irrelevant formulae are listed below.

\[
sin(2x) = 2 \sin(x) \cos(x).
\]

\[
\cos(2x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x).
\]