Examination II for PHYS 4310/5310, Spring 2005

1. Two particles have total spin quantum numbers $s_1 = 5/2$ and $s_2 = 1$, respectively. The two spins may be added to obtain new spin states which are simultaneous eigen-states of the total spin $\hat{\mathbf{S}} \equiv \hat{\mathbf{S_1}} + \hat{\mathbf{S_2}}$ and $\hat{\mathbf{S_z}}$.

(a) List the possible values of the total spin quantum numbers s, of states obtained by adding the two spins. (1 point)

(b) List all the possible values of the quantum numbers m, corresponding to the z component of the new composite spins. (1 point)

(c) List each of the possible spin states in the form $|s, m\rangle$. (3 points)

2. An electron in the hydrogen atom has a wavefunction $\Psi(\vec{\mathbf{r}}, t) = (\Psi_{1,0,0} + \Psi_{5,1,0})/\sqrt{2}$. Find $\langle \sin(\theta) \rangle$, where θ is the polar angle of the position vector $\vec{\mathbf{r}}$ of the electron. (4 points)

3. A particle lies in the state $|n\rangle$, which is the nth energy eigen-state of a one dimensional harmonic oscillator with eigen energy E_n, where n is a non-negative integer. The classical vibrational frequency of the particle is ω.

(a) Write the quantum Hamiltonian $\hat{\mathbf{H}}$ for the system in the position basis in terms of the mass m of the particle, ω, and $\hbar$. You need not derive it. (2 points)

(b) What is the expectation value of $\hat{\mathbf{H}}$, in terms of $\hbar$, ω and n. (2 points)

(c) Calculate the expectation value of the kinetic energy of the particle. (3 points)

4. A particle lies in a potential $V(\vec{\mathbf{r}}) = V(r)$, where $\vec{\mathbf{r}}$ is the position vector of the particle. In spherical polar coordinates $\vec{\mathbf{r}} = (r, \theta, \phi)$. One or more of the following options can be used to complete the statement below. List all the options which will form a true statatement. (3 points)

Statement: We can simultaneosly measure any of the physical properites of the particle corresponding to

(a) L_z and L^2 but not E.

(b) L_y and L^2 but not L_z.

(c) L_z and L_x but not L^2.

(d) L_x and E.

(e) L_z, L^2, and E.

5. An electron in a hydrogen atom was measured to have a total energy $E = E_1/25$ where E_1 is its ground state energy. The z component of its orbital angular momentum was measured to be $-3\hbar$. Write a general expression for its wavefunction, in terms of the energy eigen-functions $\Psi_{n,\ell,m}$ of the problem. (1 point)

Some relevant and irrelevant formulae are listed below.

$\sin(2x) = 2\sin(x)\cos(x)$.

$\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$.
