Examination I for PHYS 4310/5310, Spring 2005

1. A free particle of mass m, is in an initial state $|\Psi(0)\rangle$, which is an eigenstate of the position operator $\hat{\mathbf{X}}$, with eigenvalue b.

(a) Write an expression for $\Psi(x,0)$. (1 point)

(b) The propogator for this problem is given by $\hat{\mathbf{U}}(t) \equiv \int_{-\infty}^{\infty} dp\ |p\rangle\langle p| \exp(f(p,t))$, where $|p\rangle$ are momentum eigenstates. What is the form of f(p,t)? (1 point)

(c) Using the forms of $\Psi(x,0)$, from part (a) and the propogator in part (b), express $\Psi(x,t)$ as a single integral over p of some function that depends on p only? You need not evaluate this integral over p. (2 points)

2. A particle has the normalized wavefunction $\Psi(x,0) = \left(\sqrt{k}\right)\exp(-k|x|)$, where $k > 0$ is a constant of appropriate dimensions.

(a) Find $\langle x\rangle$ and $\langle p\rangle$. (1 point)

(b) Find $\langle x^2\rangle$. (2 points)

(c) Find a bound on $\langle p^2\rangle$? (1 point)

3. A particle of mass m lies in an infinite square well potential, with walls at $x = 0$ and $x = a$. It has an initial wavefunction $\Psi(x,0) = A(\Psi_1(x) + 7\Psi_5(x))$. $\Psi_n(x)$, $n = 1, 2...$, are the energy eigen functions. Work out all parts only at time t = 0.

(a) Find A? (1 point)

(b) If a measurement is conducted for the particle energy what values may be obtained? Express your answer in terms of m, $\hbar$, and a. (1 point)

(c) If the energy is measured twice in succession, derive the probability of finding an energy $(5\pi\hbar)^2/(2ma^2)$, on the second measurement. Assume that you do not know the outcome of the first measurement. (2 points)

(d) After this second measurement where an energy $(5\pi\hbar)^2/(2ma^2)$ was obtained the position of the particle x was measured. What would be the probability of getting a value in the range $x_0 \pm (dx/2)$. (1 point)

4. A particle lies in the potential V(x) given by

$$
\begin{aligned}
V(x) &= V_1, \quad \forall x < 0, \\
&= 0, \quad \forall 0 \le x \le a \\
&= V_2, \quad \forall x > a
\end{aligned}
$$

V_1, V_2, and a are postive constants of appropriate dimensions with $V_1 < V_2$. The energy E of the particle is such that $0 < E < V_1$.

(a) Find the general solution to the time independent Schrödinger equation in the three regions. Retain all the constants of integration. (4 points)

(b) Identify any unphysical solutions if they exist in the general solution. Give a reason to say why they are unphysical. If all solutions are physical then say what conditions this solution satisifies for it to be physical. (3 points)