Examination II for PHYS 6220/7220, Fall 2007

1. Consider a particle of mass m which is constrained to move on the surface of a sphere of radius R. There are no external forces on the particle other than those that maintain the constraint.
(i) Choose and appropriate set of coordinates and write the Lagrangian of the system. (**1 point**)
(ii) Derive the Hamiltonian of the system from this Lagrangian. (**1 point**)
(iii) State in words and then write expressions for all the constants of motion. (**2 points**)
(iv) From these prove that the particle can only move on a great circle of the sphere. (**1 point**)

2. Consider two unit vectors $\mathbf{n_A}$ = (0, 0, 1) and $\mathbf{n_B} = (1,1,0)/\sqrt{2}$. A vector is first reflected in a plane normal to $\mathbf{n_A}$. After that the new vector is reflected in a plane normal to $\mathbf{n_B}$.
(i) Find all elements of the matrix **A** corresponding to the first reflection. (**1 point**)
(ii) Find all elements of the matrix **B** corresponding to the second reflection. (**1 point**)
(iii) If the resulting total change of the vector is represented by a matrix **R** then find all its elements. (**1 point**)
(iv) Does **R** correspond to a pure rotation? Justify your answer. If **R** does indeed correspond to a pure rotation find the corresponding angle of rotation. If your answer is negative then does **R** correspond to a pure reflection? If so, what is the unit normal to the plane of reflection? (**2 points**).

3. A particle of mass m, and magnitude of angular momentum ℓ, moves in a central force potential $V(r) = -k/(2r^2)$, where k is a positive constant. Assume now that $(mk)/\ell^2 < 1$. It is known that r = b is the perigee of the orbit.
(a) Use the appropriate form of the orbit equation to obtain the most general form of the equation of the orbit $r = r(\theta)$. (**2 points**)
(b) Analyze the motion to eliminate the unknown constants and obtain limits on θ. (**2 points**)
(c) Find $\theta = \theta(t)$ and $r = r(t)$. (**2 points**)

4. A particle of mass m moves in a potential V(r). The particle is found to have a maximum distance b and a minimum distance a, from the center of force in one period of its orbit. Express all answers only in terms of m, a, b, V(a) and V(b).
(a) Find the angular momentum of the particle. (**1 point**)
(b) Find the total energy of the particle. (**1 point**)
(c) Find the speed of the particle when r = a and r = b. (**2 points**)