Examination I for PHYS 6220/7220, Fall 2007

1. Consider a Lagrangian $L = L(\{q_i\}, \{\dot{q}_i\}, t)$ and a function $F = F(\{q_i\}, t)$. Let the Hamiltonian be defined as $H(\{q_i\}, \{p_i\}, t) \equiv \sum_{i=1}^{N} \dot{q}_i p_i - L$. Define a new Lagrangian

$L_n \equiv L + \dot{F}$ and a new Hamiltonian, $K = K(\{q_i\}, \{P_i\}, t)$ such that $K \equiv \sum_{i=1}^{N} \dot{q}_i P_i - L_n$ where a dot signifies a full time derivative, i.e., $\dot{a} \equiv \frac{da}{dt}$.

(i) Calculate dK in two different ways. Then define P_i appropriately in terms of L and F in analogy to the definition of p_i so that dK has the correct form.

(ii) Find a simple expression for $\frac{\partial K}{\partial P_i}$, in terms of partial/full time derivatives of q_i or P_i.

(iii) Find a simple expression for $\frac{\partial K}{\partial q_i}$, in terms of partial/full time derivatives of q_i or P_i.

(iv) Find a simple expression for $\dot{K}$ in terms of partial/full time derivatives of L & F.

(v) Are all three Hamilton equations for K obeyed? Comment on any special equation that F follows.

(**5 points**)

2. A hemisphere of mass M and radius a rests with its flat surface on a frictionless horizontal plane as shown in Fig. 1. A mass m initially at rest at the top of the frictionless hemisphere looses its position of unstable equilibrium at time t = 0 and starts sliding on the surface of the hemisphere under the influence of gravity. Magnitude of the acceleration due to gravity is g. The X and Z axis are as shown. The radial vector to mass m from the center of the hemisphere makes an angle θ with the Z axis, at time t, as shown.

(a) Define clearly an appropriate set of generalized coordinates. Use these to obtain the Lagrangian of the system. (**2 points**)

(b) Use a Lagrange multiplier λ associated with the constraint that m moves over the hemisphere and write Langrange equations of motion. (**2 points**)

(c) Write an expression for λ purely as a function of one generalized coordinate and its derivatives with time. (**2 points**)

(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)

(e) Eliminate the time derivatives in part (c) to express λ purely in terms of one generalized coordinate. (**2 points**)

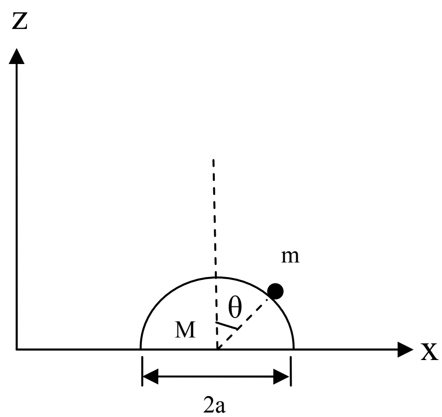

Fig. 1

3. Consider a system of two masses connected by a mass-less string of length ℓ with m_2 constrained to stay on the surface of an upright cone of half-angle α, as shown, and m_1 hanging freely inside the cone. The string passes through a hole at the top of the cone as shown in Fig. 2. The magnitude of the acceleration due to gravity is g. Neglect friction. Use spherical polar coordinates to solve this problem with the origin at the hole.

(a) State the degrees of freedom. Choose an appropriate set of generalized coordinates for the problem. (1 **point**)

(b) Write an expression for the kinetic energy for the system using these coordinates. (1 **point)**

(c) Write an expression for the potential energy of the system. (1 **point**)

(d) State all the constants of motion in words and then write expressions for all the constants of the problem in terms of generalized coordinates and velocities. (2 **points**)

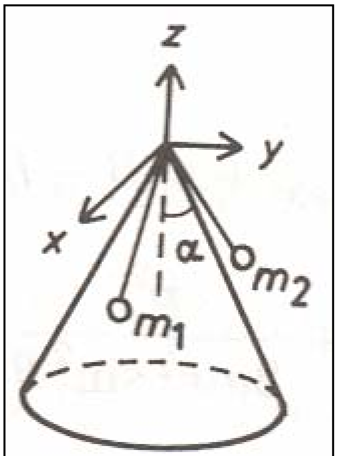

Fig. 2