Final Examination for PHYS 6220/7220, Fall 2007

1. Four equal masses each of value m, lie at positions (x, y, z) = (b, 0, 0), (-b, 0, 0), (0, 2b, 0) and (0, -2b, 0).
(a) Find the inertia tensor, using the Cartesian reference system in which these coordinates have been given. [**3 points**]
(b) Express a unit vector **n** in the same Cartesian coordinate system such that it makes equal angles with all three (X, Y, and Z) axes. [**1 point**]
(c) If at a particular instant the angular velocity $\boldsymbol{\omega} = \omega\mathbf{n}$ of this system then find the angle that the instantaneous angular momentum **L** makes with **n**. [**2 points**]
(d) Find the moment of inertia about an axis that passes through the origin and is parallel to **n**. [**2 points**]

2. A plumb line in Toledo may be modeled as a simple pendulum at rest. Let the mass of the bob be m. Choose an appropriate coordinate system at the center of the earth. Answer all questions in this coordinate system. The radius of the earth is R, and acceleration due to gravity is g and the rotational angular frequency of the earth is ω. Toledo is at a latitude θ.
(a) Find the total force on the bob due to gravity and the rotation of the earth. [**4 points**]
(b) Find the angles that the plumb line makes with axes of your coordinate system. [**2 points**]
(c) What would be the answer to part (b) if we ignored the rotation of the earth. [**1 point**]

3. A particle is constrained to move on a right circular cylinder which has an equation given by $x^2 + y^2 = b^2$, where b is the radius of the cylinder. There is no gravity in this problem. It is subject to a potential V(**r**) = -k/r, (k > 0), where **r** is its position vector and r = |**r**|. Choose an appropriate coordinate system suitable to the symmetry of the problem. Do not use Lagrange multipliers.
(a) Write the Lagrangian for the system. [**2 points**]
(b) Find the Euler Lagrange equations. [**2 points**]
(c) Solve these to obtain the most general solution to the problem. You may leave one of the solutions in the form $t = \int_0^q f(q)dq$, where q is one of the generalized coordinates, and t is the time. However, you must derive the precise form of f(q). [**2 points**]
(d) Eliminate the unknown constants if it is known that the initial position of the particle was at x = b, z = 0 and its velocity had a component v_1 parallel to z-axis and v_2 perpendicular to it. [**1 point**]

4. Two equal masses each of value m are connected to two identical springs with spring constant κ. As shown in the Fig. One end of one of the springs is fixed at point A. There is no gravity in this problem.
(a) Write the Lagrangian of the system by choosing an appropriate set of coordinate axes. Define and explain your coordinates clearly. [**2 points**]
(b) Find the eigenfrequencies for small oscillations of this system about its equilibrium configuration. [**2 points**]

(c) Find the corresponding eigenvectors. [**2 points**]
(d) Find the most general solution. [**1 point**]
(e) Show with arrows the relative phases of the particles in the normal modes. [**1 point**]

Fig. for problem 4