Examination II for PHYS 6220/7220, Fall 2006

1. A one dimensional simple harmonic oscillator has mass m, and generalized canonical coordinates q and p, and angular vibrational frequency ω, and Hamiltonian H(q, p).

(a) Evaluate [u, H] where $u = -i\omega t + \ln(p + i m\omega q)$ and $i \equiv \sqrt{-1}$. (**2 points**)

(b) Use result in part (a) to obtain du/dt. Comment on your result. (**2 points**)

(c) Express H as H = H(u) and other known constants. (**1 point**)

2. Two successive rotations are performed on a rigid body with a common fixed point on the body for both rotations. Each rotation is through π radians. The two rotation axes are defined by unit vectors in the laboratory Cartesian coordinate system given by $\mathbf{n_1} = (1,1,0)/\sqrt{2}$, and $\mathbf{n_2}$ = (0, 0, 1) respectively.

(a) Find all elements of the matrix **A** corresponding to the first rotation. (**1 point**)

(b) Find all elements of the matrix **B** corresponding to the second rotation. (**1 point**)

(c) If the resulting net displacement of the body is represented by a matrix **R** then find all its elements. (**2 points**)

(d) Find the resulting angle of rotation as if only one effective rotation was performed on the body through only one axis. (**1 point**)

3. A particle of mass m approaches a center of force from a far away distance with initial speed v_0 and impact parameter b. The center of force exerts a force on the particle corresponding to the potential $V(r) = -k/r^4$, where r is the distance of the particle from the center of force. Express all answers in terms of the known constants, m, k, v_0, and b.

(a) Find the total energy E of the particle. (**1 point**)

(b) Find the magnitude of its angular momentum ℓ calculated with respect to the center of force. (**1 point**)

(c) Find the distance of closest approach c of the particle to the center of force. (**2 points**)

(d) Find its angular speed at the distance of closest approach. (**2 point**)

4. Consider the motion of a particle of mass m and magnitude of angular momentum ℓ, moving in a potential $V(r) = -[(k/r) + (k'/r^3)]$, where r is the distance of the particle from the origin.

(a) Find the radius r_0 of a circular orbit for the particle in terms of k, k', m, and ℓ. (**2 points**)

(b) Find the condition for stability of this orbit relating r_0 with k and k'. (**2 points**)