Examination I for PHYS 6220/7220, Fall 2006

1. For many physical situations one encounters a general linear differential equation of constraint in the set of coordinates $\{x_i, i = 1, 2, \ldots n\}$ of the form $\sum_{i=1}^{n} g_i(x_1, x_2, \ldots x_n) dx_i = 0$.

A constraint of this type may be integrated if an integrating function $f = f(x_1, x_2, \ldots x_n)$ is found such that it obeys $\frac{\partial(f g_i)}{\partial x_j} = \frac{\partial(f g_j)}{\partial x_i}, \quad \forall i, j$.

In a particular problem we obtain the following set of coordinates: x, y, θ, and ϕ. These follow a constraint equation: $dx - a \sin\theta \, d\phi = 0$.

For this equation where 'a' is a constant, find the factor f, if it exists and thus integrate the equation. If it does not exist then prove rigorously that f = 0. (**4 points**)

2. A Hamiltonian is given by H(x,p) = (cp)/(ax), were a and c are constants and x >0.
(a) Solve the Hamilton equations of motion to obtain x = x(t) and p = p(t). (**4 points**)
(b) Obtain the Lagrangian for the system $L = L(x, \dot{x}, t)$. (**1 point**)

3. A block of mass m_1 has attached to it a string of fixed length ℓ and of negligible mass. At the other end of the string is a freely hanging bob of mass m_2. The block is restricted to move only along a horizontal straight line as shown in the figure. Both masses and the string are restricted to move in a single plane at all times.
(a) Define clearly an appropriate set of generalized coordinates. Use these to obtain the Lagrangian of the system. (**2 points**)
(b) Use a Lagrange multiplier λ associated with the constraint that the string is of fixed length and write the Langrange equations of motion. (**2 points**)
(c) Solve for λ purely as a function of the generalized coordinates. (**2 point**)
(d) State in words the constants of motion in this problem. Write expressions for these constants in terms of the generalized coordinates and generalized velocities. (**2 points**)
(e) For what value of the generalized coordinates will λ be zero? What does it signify? (**1 point**)

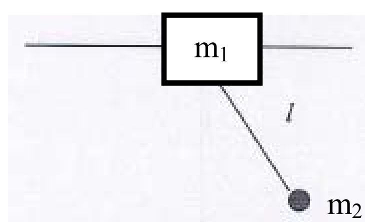

Fig. 1