Final Examination for PHYS 6620/7220, Fall 2004

1. Three point masses each of mass m, are located at (a,0,0), (0,a,2a), and (0,2a,a) in a Cartesian coordinate system.

(a) Find the components of the the interita tensor with respect to the origin, in units of ma^2. (3 points)

(b) Write the appropriate equation $D = 0$, where D is the determinant containing the eigenvalue λ (as yet undetermined) to diagonalize this intertia tensor. (1 point)

(c) Taking a common factor out from one of the rows or columns of D will quickly yield one value of λ. Find that value. (1 point)

(d) Now find the remaining two eigenvalues of the tensor. (2 point)

(e) Find the eigenvectors. (3 points)

2. A circular disc in a horizontal plane rotates with an angular speed ω, about an axis through its center and perpendicular to the plane of the disc. A hockey puck is stuck at rest at a position vector $\vec{\mathbf{r}}_0$ from the center of the disc. Define a convenient Cartesian frame of reference C fixed to the disc. Parts (a) and (b) below are independent of each other. Express answers to both parts in terms of m, ω, v_0, and appropriate Cartesian unit vectors of C.

(a) Find the effective force on the puck in the frame C, after it is unstuck and given a radial push with instantaneous speed v_0 with respect to the disc. (2 points)

(b) Find the effective force on the puck in the frame C, after it is unstuck and given a push in the horizontal plane perpendicular to the radial direction with instantaneous speed v_0. (2 points)

3. A particle of mass m moves in an elliptical orbit in a central force field of inverse square force law, i.e. $f(r) = -k/r^2$, $k > 0$. The ratio of the maximum angular speed to the minimum angular speed of the particle is n.

(a) Find the minimum and maximum angular speeds of the particle in terms of m, the eccentricity of the ellipse ϵ, the semi-major axis a, and magnitude of the angular momemtum ℓ. (2 points)

(b) Find the eccentricity of the ellipse ϵ as a function purely of n. (1 point)

4. A spherical pendulum consists of a point mass m suspended by a rigid massless rod of length ℓ.

(a) Draw a Fig. and show the appropriate generalized coordinates for the system. (1 point)

(b) Write the Lagrangian for the system in these generalized coordinates. (1 point)

(c) Write expressions for the constants of the motion in terms of the generalized coordinates. (2 points)

(d) Using one of the constants eliminate one of the generalized coordinates from L thus reducing the problem to that of a one dimensional system. (1 point)

5. Two masses each of mass m, are connected to three springs and two fixed walls as shown in the figure below. The springs attached to the walls each have spring constant κ and the spring connecting the two particles has spring constant κ_{12}. All three springs have equilibrium lengths ℓ.

(a) Use appropriate generalized coordinates and find the Lagrangian for the system. (2 points)

(b) Find the eigenfrequencies for small oscillations of the two masses. (2 points)

(c) Find the corresponding properly normalized eigenvectors. (2 points)

(d) Depict with arrows on the mass points the motion corresponding to the eigenvectors. (1 point)

(e) Find the general solution to the problem. (1 point)