Examination II for PHYS 6620/7220, Fall 2005

1. We are given a Hamiltonian

$$H(q_1, q_2, p_1, p_2) \equiv \sum_{n=1}^{2} [q_n p_n][(-1)^{1+n}]. \tag{1}$$

A physical quantity g is defined as $g \equiv q_1 q_2$.

(a) Evaluate $[g, H]$. (2 points)

(b) Using result in (a) find $\dot{g}$. Comment on your result (1 point)

(2) Using the fundamental Poisson brackets find the values of α and β for which the equations

$$Q = q^\alpha \cos(\beta p) \quad \text{and} \quad P = q^\alpha \sin(\beta p) \tag{2}$$

represent a canonical transformation. (2 points)

3. It is known that for any nonsingular matrix $\overline{\overline{\mathbf{M}}}$ that the inverse and transpose operations commute, i.e. $\left(\overline{\overline{\mathbf{M}}}^{-1}\right)^T = \left(\overline{\overline{\mathbf{M}}}^{T}\right)^{-1}$. A nonsingular matrix is one whose determinant is not zero.

(a) $\overline{\overline{\mathbf{A}}}$ is an antisymmetric real matrix. How many of the nine elements $\{a_{ij}, i, j = 1, 2, \text{ and } 3\}$ of this matrix can be independently chosen? (1 point)

(b) Show with an explicit calculation using the result of part (a) that $\overline{\overline{\mathbf{I}}} \pm \overline{\overline{\mathbf{A}}}$ are nonsingular, where $\overline{\overline{\mathbf{I}}}$ is the identity matrix. You may prove the result with just one sign and only argue what effect the sign change would have on your calculation. This will save you some time. (2 points)

(c) Use results in parts (b) to show that $\overline{\overline{\mathbf{B}}} \equiv \left[\overline{\overline{\mathbf{I}}} + \overline{\overline{\mathbf{A}}}\right] \cdot \left[\left(\overline{\overline{\mathbf{I}}} - \overline{\overline{\mathbf{A}}}\right)^{-1}\right]$ is orthogonal. (2 point).

4. A particle of mass m, and magnitude of angular momemtum ℓ, moves in a central force potential $V(r) = -k/(2r^2)$, where k is a postive constant. Assume now that $\ell^2/(mk) = 1$.

(a) Use the appropriate form of the orbit equation to obtain the most general form of the equation of the orbit $r = r(\theta)$. (3 points)

(b) Find the dependence of θ on t assuming $\theta = 0$ at t = 0. (3 points)

5. A particle of mass m, and magnitude of angular momemtum ℓ, moving in a central potential $V(r) = -k/r$, follows an elliptical orbit with eccentricity ϵ, period τ and semi-major axis a.

(a) Use conservation of energy to obtain the radial velocity of the particle as a function of its energy E, ℓ, m, and r. (2 points)

(c) Convert this expression for the radial velocity so that it involves only terms in a, ϵ, τ, and r. (2 points)