Examination II for PHYS 6620/7220, Fall 2004

1. A one dimensional system has a Hamiltonian

$$H(q,p,t) = \frac{p^2}{2} - \frac{1}{2q^2}. \tag{1}$$

Define $D_1 \equiv (pq)/2$, $D_2 \equiv tH$, and $D \equiv D_1 - D_2$.

(a) Calculate the Poisson brackets $[D_1, H]$ and $[D_2, H]$. (3 points)

(c) Using the above results calculate $\dot{D} \equiv \frac{dD}{dt}$ (2 points)

2. A matrix $\overline{\overline{\mathbf{A}}}$ is a rotation matrix corresponding to a rotation of 180° about an axis which is along the unit vector $\hat{\mathbf{n}}$. The identity matrix is $\overline{\overline{\mathbf{I}}}$. Define

$$\overline{\overline{\mathbf{P}}}_{\pm} \equiv \frac{(\overline{\overline{\mathbf{I}}} \pm \overline{\overline{\mathbf{A}}})}{2}. \tag{2}$$

(a) Give a physical argument using a figure to deduce the effect of operating $\left(\overline{\overline{\mathbf{A}}}\right)^2$ on any arbitrary vector. (2 points)

(b) Use the result in (a) to write a simple matrix expression for $\left(\overline{\overline{\mathbf{A}}}\right)^2$. (1 point)

(c) Using result from (b) we may express write $\left(\overline{\overline{\mathbf{P}}}_{\pm}\right)^2 = a\left(\overline{\overline{\mathbf{P}}}_{\pm}\right) + b\left(\overline{\overline{\mathbf{I}}}\right)$. Find the value of the constants a and b. (2 points).

3. A particle of mass m, and magnitude of angular momemtum ℓ, moves in a central potential V(r). The equation of its orbit is $r = a\exp(k\theta)$, where a and k are constants.

(a) Use the appropriate form of the orbit equation to obtain V(r). (3 points)

(b) Find the dependence of θ on t, assuming $\theta = 0$ at $t = 0$. (3 points)

4. A particle of mass m, and magnitude of angular momemtum ℓ, moves in a central potential $V(r) = (kr^2)/2$.

(a) What is the effective one dimensional potential $V_e(r)$ for this problem. (1 point)

(b) What is the radius of a circular orbit in such a potential? (2 points)

(c) What is the total energy of the particle in such an orbit? (1 point)