Examination II for PHYS 6620/7220, Fall 2004

1. A one dimensional system has a Hamiltonian

\[ H(q,p,t) = \frac{p^2}{2} - \frac{1}{2q^2}. \]  \hspace{1cm} (1)

Define \( D_1 \equiv (pq)/2, D_2 \equiv tH, \) and \( D \equiv D_1 - D_2. \)
(a) Calculate the Poisson brackets \([D_1, H]\) and \([D_2, H]\). (3 points)
(c) Using the above results calculate \( \dot{D} \equiv \frac{dD}{dt} \) (2 points)

2. A matrix \( \overline{A} \) is a rotation matrix corresponding to a rotation of 180° about an axis which is along the unit vector \( \hat{n} \). The identity matrix is \( \overline{I} \). Define

\[ \overline{P}_\pm \equiv \frac{(\overline{I} \pm \overline{A})}{2}. \]  \hspace{1cm} (2)

(a) Give a physical argument using a figure to deduce the effect of operating \( (\overline{A})^2 \) on any arbitrary vector. (2 points)
(b) Use the result in (a) to write a simple matrix expression for \( (\overline{A})^2 \). (1 point)
(c) Using result from (b) we may express write \( (\overline{P}_\pm)^2 = a(\overline{P}_\pm) + b(\overline{I}) \). Find the value of the constants \( a \) and \( b \). (2 points).

3. A particle of mass \( m \), and magnitude of angular momentum \( \ell \), moves in a central potential \( V(r) \). The equation of its orbit is \( r = a \exp(k\theta) \), where \( a \) and \( k \) are constants.
(a) Use the appropriate form of the orbit equation to obtain \( V(r) \). (3 points)
(b) Find the dependence of \( \theta \) on \( t \), assuming \( \theta = 0 \) at \( t = 0 \). (3 points)

4. A particle of mass \( m \), and magnitude of angular momentum \( \ell \), moves in a central potential \( V(r) = (kr^2)/2 \).
(a) What is the effective one dimensional potential \( V_e(r) \) for this problem. (1 point)
(b) What is the radius of a circular orbit in such a potential? (2 points)
(c) What is the total energy of the particle in such an orbit? (1 point)