Examination I for PHYS 6620/7220, Fall 2004

T1

1. It is given that $F = F(\{q_i\}, t)$. A dot over a symbol represents a total time derivative.

(a) Write an expression for $\dot{F}$. (1 point)

(b) Using result from (a) evaluate $\frac{\partial \dot{F}}{\partial \dot{q}_i}$. (2 points)

(c) Now use result from (b) to show that $\dot{F}$ obeys Lagrange's equations. (1 points)

2. A Lagrangian of coordinate q is given by

$$L = \left(\frac{e^{\alpha t}}{2}\right)\left(m\dot{q}^2 - kq^2\right). \tag{1}$$

where m, k, and α are constants.

(a) Is the corresponding Hamiltonian H(q,p,t) conserved? Give a reason for your answer. You need not obtain H. (1 point)

(b) Make a substitution $s = (e^{\frac{\alpha t}{2}})q$, to obtain $L(s, \dot{s}, t)$. (2 points)

(c) Find the new Hamiltonian. (3 points)

(d) Is the corresponding Hamiltonian conserved? Give a reason for your answer. (1 point)

3. The tip of an inverted cone is fixed to the surface of the earth with its symmetry axis along the vertical z axis. Its inside surface has an equation given by $r = z$ and $z \geq 0$ where r is the distance from the z axis. A particle of mass m moves with constant angular speed ω on the inside surface of this cone. Assume there is no friction.

(a) Use an appropriate set of coordinates to obtain the Lagrangian of the system. (2 points)

(b) Use a Lagrange multiplier λ associated with the constraint that the particle moves on the inside surface of the cone and write down the Lagrange equations of motion. (2 point)

(c) Solve for λ in terms of the distance of the particle from the z axis and other constants. (1 point)

(d) Under what conditions will the particle remain at constant height? (1 point) What is the value of λ when this happens? (1 point) In this particular situation what are the constants of motion? (2 points).