Final Examination for PHYS 6220/7220, Fall 2005

1. Three point masses each of mass m, are located at (a, 2a, 0), (0, a, 2a), and (2a, 0, a) in a Cartesian coordinate system.

(a) Find the components of the the interita tensor with respect to the origin, in units of ma^2. (3 points)

(b) Write the appropriate equation D = 0, where D is the determinant containing the eigenvalue λ (as yet undetermined) to diagonalize this intertia tensor. (1 point)

(c) Use matrix row operations to take some common factors out of this determinant. This should yield two values of λ easily. Then find the remaining value. (3 point)

(d) Find the eigenvectors. (3 points)

2. A simple pendulum of mass m is hanging at rest in Toledo which has a latitude θ. The magnitude of the acceleration due to gravity is g. The radius of the earth is R and the rotational speed of the earth around its own axis is ω.

(a) Draw a figure showing the earth and pendulum. Choose a convenient set of Cartesian axes at the center of the earth to answer the question in part (b). Clearly label important points on the figure. (1 point)

(b) Find the effective force acting on the bob due to gravity and the rotation of the earth. Describe all components in terms of unit vectors along the Cartesian axes of part (a) and other given quantities. That the bob is at rest may simplify your answer. (3 points)

3. A particle of mass m is in an elliptical orbit of ecceentricity ϵ in a central force potential $V(r) = -k/r$, $k > 0$. The ratio of the maximum linear speed to the minimum linear speed of the particle is n.

(a) Use conservation of energy to relate the minimum and maximum linear speed of the particle to its maximum or minimum distance from the center of force. Use these relationships to find the minimum and maximum linear speeds of the particle in terms of m, ϵ, k, and the semi-major axis a. (2 points)

(b) Find n purely as a function of the ϵ. (1 point)

4. A spherical pendulum consists of a point mass m suspended by a rigid massless rod of length ℓ. Note that this pendulum is not restricted to be in a plane.

(a) Draw one or more figures and show the appropriate generalized coordinates for the system. (1 point)

(b) Write the Lagrangian for the system in these generalized coordinates. (1 point)

(c) Write expressions for the constants of the motion in terms of the generalized coordinates. (2 points)

(d) Using one of the constants to eliminate one of the generalized coordinates from L thus reducing the problem to that of a one dimensional system. (1 point)

5. Two identical simple pendula of length b, and mass m each are coupled by a spring of force constant κ as shown in the figure. When the masses are hanging vertically the spring is at its equilibrim length ℓ. When they oscillate they make angles θ_1 and θ_2 with the vertical as shown. These pendula are restricted to move in the plane of the figure. For $\ell \ll b$ which we assume, the stretching or compression of the spring may be taken to be $b(\sin(\theta_1) - \sin(\theta_2))$.

(a) Use appropriate generalized coordinates and find the Lagrangian for the system. (3 points)

(b) Now assume that the pendula oscillations have small amplitudes and approximate the Lagrangian to treat the problem as that of coupled small oscillations. (3 points)

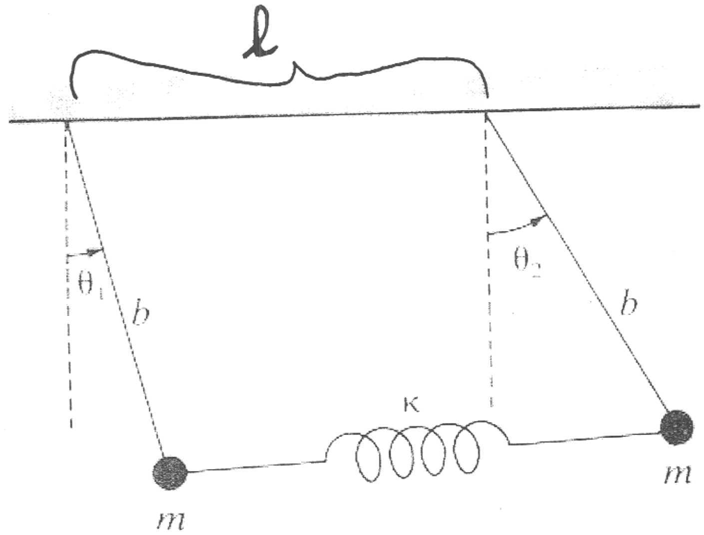
(c) Write expressions for the matrices $\bar{\bar{\mathbf{T}}}$ and $\bar{\bar{\mathbf{V}}}$ in terms of the given quantities and generalized coordinates. (2 points)

(d) Without solving the problem depict with arrows on the two masses in your answer sheets, the two normal modes you expect? (2 point)

Some relevant and irrelevant formulae

(1) $\sin(x) = \sum_{n=0}^{n=\infty} \left[\frac{(-1)^n x^{2n+1}}{(2n+1)!}\right]$ and $\cos(x) = \sum_{n=0}^{n=\infty} \left[\frac{(-1)^n x^{2n}}{(2n)!}\right]$.

(2) $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$.



l
θ1
b
θ2
b
κ
m
m