Valence electron concentration as an indicator for mechanical properties in rocksalt structure nitrides, carbides and carbonitrides

Karthik Balasubramanian a, Sanjay V. Khare b, Daniel Gall c, *

a Department of Mechanical, Nuclear and Aerospace Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, USA
b Department of Physics and Astronomy, The University of Toledo, 2801 West Bancroft Street, Toledo, OH 43606, USA
c Department of Materials Science and Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, USA

1. Introduction

Transition metal nitrides and carbonitrides are widely used as hard and protective coatings [1–10] and as diffusion barrier layers [11–14] due to their excellent thermal stability, mechanical properties, and corrosion resistance [1–4,10,15–18]. Many transition metal nitrides and carbonitrides crystallize in the rocksalt structure, including ScN [19], TiN [20], HfN [3], ZrN [21], VN [22], NbN [23], TaN [18], CrN [24], MoN [25], WN [26], FeN [27], NbC 1−x N x [2], TaC 1−x N x [28], VC 1−x N x [29], HfC 1−x N x [30], ZrC 1−x N x [31] and TiC 1−x N x [32]. The early transition metal nitrides, carbides and carbonitrides, particularly with metal elements of group 4, are well studied and have excellent mechanical properties which are predicted to further improve with an increasing number of valence electrons [33,34], resulting in increased ductility and toughness when moving to the right in the periodic table [35–39]. However, large electron concentrations can lead to negative C 44 values and a corresponding mechanical instability, as reported for example for rocksalt structure WN [40,41] and MoN [25,40,42]. This instability has been attributed to an increasing overlap and constructive interference of d−t 2g orbitals upon shear, leading to an energy reduction during shear deformation and a mechanical instability of the cubic structure [41,43–45]. Correspondingly, the mechanical properties of rock-salt structure transition metal nitrides vary widely [15,17,46]. For example, the elastic modulus E decreases from theoretically calculated 450 GPa for TiN [33,47] to −285 GPa for WN [41], with intermediate values of 370 GPa for ZrN [33], 386 GPa for VN [35], 283 GPa for V 0.5 W 0.5 N [37], and experimental
values measured by nanindentation of 400 GPa for HfN [3] and 273 GPa for NbN [23]. Combining binary nitrides to form ternary and quaternary nitrides and/or extending the materials system to carbonitrides provides ample opportunities to optimize strength, ductility and toughness for a specific application. For this purpose, it is desirable to have a preliminary indicator for mechanical properties of transition metal nitrides and carbonitrides which provides guidance in the process of alloy design.

The valence electron concentration (VEC) is defined as the number of valence electrons per formula unit. This parameter has been reported to be a significant indicator of the structural, thermodynamic and mechanical properties of binary and ternary compounds in various materials systems, and has been used to predict the phase stability and relative formation enthalpy in high entropy alloys [48], the critical temperature of high temperature superconductors [49], elastic and electronic properties of alkaline earth titan oxides [50], bulk moduli for transition metal nitrides in the pyrite and fluorite phase [51], the stability and electronic structure in Mo-S-I nanowires [52], and the formation enthalpy in Y carbonitrides and boronitrides [53]. Similarly, tuning of the VEC to tailor mechanical properties has also been employed to transition metal carbonitrides. In particular, VEC = 8.4 has been predicted to yield a maximum strength in TiC1-xNx alloys [44,45,54], while increasing VEC above 8.4 in transition metal nitrides and carbonitrides has been predicted to increase ductility and toughness [35,38,55], which may be associated with an increased overlap of d–f28 orbitals [35,38,56] and the reported decrease in the shear modulus [57]. VEC tuning has also been employed to enhance hardness in transition metal carbides and the Nb4Zr1–xN x system by promoting nucleation of stacking faults and phase boundaries during synthesis and hindering dislocation motion across these faults and phase boundaries [58,59] and enhancing ductility of pseudo binary cubic Ti2N1–xN x alloys [60]. Furthermore, VEC tuning has also been employed to predict enhanced mechanical properties of Mo–Al–Cr–N alloys by varying the relative compositions of the components [61] and increasing the thermal stability of Ti–Al–N alloys by suitable Ta alloying [62]. These previous studies motivate a systematic study on the trends of all the elastic constants, moduli and hardness of transition metal carbonitrides in the rocksalt structure as a function of VEC, focusing particularly on brittle-to-ductile transitions and mechanical stability, with the overall goal to provide guidelines for designing tougher alloys.

In this work, we use ab initio calculations to identify trends in the mechanical properties of rock-salt structure transition metal nitrides and carbonitrides from groups 4 to 12. The valence electron concentration is demonstrated to be a key indicator for mechanical properties. Increasing the VEC leads to a decrease in Pugh’s ratio and an increase in Poisson’s ratio, indicating an increasing ductility with increasing VEC, and a brittle to ductile transition at VEC ~ 10. The shear elastic constant C44 decreases with increasing VEC, indicating a transition to mechanical instability for VEC > 10.6. Similarly, the average calculated isotropic elastic modulus decreases from 426 to ~83 GPa and the hardness decreases from 25 to 2 GPa with the VEC increasing from 8 to 11.

2. Computational procedure

First-principles density functional calculations were performed using the Vienna ab initio simulation package (VASP) [63,64], employing periodic boundary conditions, a plane wave basis set, the Perdew–Burke–Ernzerhof (PBE) generalized gradient approximation exchange-correlation functional [65], and the projector-augmented wave method [66]. All computational parameters were chosen such that calculated total energy differences were converged to within 1 meV/atom. This included a 500 eV cut-off energy for the plane-wave basis set expansion and a Γ-centered $20 \times 20 \times 20$ k-point grid for conventional unit cells containing 8 atoms for binary nitrides, carbonitrides and boronitrides. Special quasi-random structures (SQS) [67] were constructed for two ternary systems (Ti0.25W0.75N) and (W0.25Ni0.75N) using unit cells containing 32 cations and 32 anions, to check if random occupation of lattice sites would affect the general trends obtained using the 8 atom conventional unit cells. Pseudopotentials were chosen, following the recommendation by VASP for relatively accurate calculations, such that the following electrons were explicitly calculated (i.e. not included in the pseudopotential): 3s, 3p, 3d and 4s for the elements Ti and V; 3p, 3d and 4s for Cr and Mn; 3d and 4s for Fe to Mn; 4s, 4p, 4d and 5s for Zr to Mo; 4p, 4d and 5s for Tc to Rh; 4d and 5s for Pd to Cd; 5p, 5d and 6s for Hf to W; 5d and 6s for Re to Hg. Atomic positions and lattice parameters were relaxed until an energy convergence of $10^{-4}$ eV/atom was achieved. This was done by multiple calculations for each atomic configuration, where atoms were relaxed at a fixed lattice parameter that was varied for the different calculations by 0.01 Å increments. Subsequently, the relaxed lattice constant and corresponding energy was determined by fitting the calculated energy vs lattice parameter with a second order polynomial. The three independent elastic constants for cubic systems, C11, C12 and C44 were determined by applying a set of strains to the conventional unit cell, and fitting second order polynomial functions to the calculated elastic energy vs strain. In particular, C11 was determined from tensile and compressive ±1–3% strains along [001], C12 was calculated from the bulk modulus B using $C12 = (3B - C11)/2$ [68], where B was obtained from a fit of the calculated energy vs lattice constant using the Murnaghan equation of state, and C44 was determined using a 1–5% shear strain along [100]. For structures, which do not truly exhibit cubic symmetry, such as ternary systems, the elastic constants were averaged over the directions [69]. That is C11, C12 and C44 was averaged to yield an average tensile elastic constant $C_{T11}$, Similarly, the shear elastic constants C44, C55 and C66 were averaged to yield an average shear elastic constant $C_{S11}$. We note that these ternary systems could be referred to as pseudobinary alloys, as anti-site substitutions were not considered such that metal atoms always occupy cation sublattice sites while nitrogen and carbon atoms always occupy anion sublattice sites. We also note that DFT predictions of elastic constants have uncertainties associated with the choice of the exchange-correlation functional. The PBE functional used in our study has been reported to yield bulk moduli that deviate by 5–20% from experimental values for a large range of materials systems [70]. Correspondingly, we expect the elastic constants reported in our study to have an accuracy of 5–20%.

The isotropic elastic modulus E and isotropic shear modulus G were determined from the calculated elastic constants using Hill’s approximation [71] calculated from Voigt and Reuss methods. We note here that while the Voigt and Reuss methods yield similar values for material systems with positive elastic constants, they vary considerably for systems with negative elastic constants. Pugh’s ratio k and isotropic Poisson’s ratio ν were calculated using $k = G/B$ and $\nu = (E/2G) − 1$ [72]. Vicker’s hardness was then calculated according to Tain’s model $H_v = \frac{0.92k_0}{1.04A}$ [73]. The dynamic stability of the carbides and nitrides was investigated by calculating the phonon-dispersion curves using PHONOPY [74] and looking for imaginary phonon modes. This was done by calculating the force constants and subsequently the phonon dispersion curves by displacing atoms along high symmetry directions in a 216 atom cell ($3 \times 3 \times 3$ supercell), using a $2 \times 2 \times 2$ k-point mesh. The mechanical properties in magnetic binary nitrides, MnN, CrN, CoN, FeN and NiN, were calculated by considering paramagnetic, ferromagnetic and antiferromagnetic ordering of the ions in the [100], [110] and [111] directions by constructing $2 \times 2 \times 2$ supercells containing 32...
3. Results and discussion

Fig. 1 shows the calculated Pugh’s ratio \( k \) and Poisson’s ratio \( \nu \) for rock-salt structure binary carbides and nitrides of transition metals from groups 4–12. Open and closed symbols denote carbides and nitrides, respectively, while the plotted symbol shape and color specify the row and column of the metal in the periodic table. Pugh’s and Poisson’s ratios are determined from the calculated elastic constants, as described in Section II. They are plotted because they are commonly used as an empirical indicator for a materials ductility \[72\]. More specifically, a material is expected to be ductile, if its Pugh’s ratio \( k \) is below a critical value of 0.5–0.6 and its Poisson’s ratio \( \nu \) is above a critical value of 0.25–0.28 \[72\]. The plot in Fig. 1(a) of Pugh’s vs Poisson’s ratio indicates a strong correlation of these two parameters, with a nearly linear decrease in \( k \) with increasing \( \nu \). Linear curve fitting provides a slope of \( -2.7 \), which is close to the average slope of \( -2.6 \) for a perfectly isotropic material in the range \( \nu = 0.2 – 0.45 \), as deduced from \( k = 1.5 \times (1-2\nu)/(1+\nu) \) and correspondingly \( dk/d\nu = -4.5/(1+\nu)^2 \) \[72\]. The dotted horizontal and vertical lines divide the plot into regions of ductile and brittle materials, using critical values of \( k = 0.5 \) and \( \nu = 0.28 \). The data points from group 4 nitrides and group 4 and 5 carbides are in the left top corner of Fig. 1(a), indicating that they are expected to be brittle. In contrast, nitrides of metals from group 5–12 and carbides of metals from group 6–12 are in the bottom right region, suggesting ductility. We attribute the increasing ductility with increasing VEC to a change from a more covalent bonding character to more metallic bonding. More specifically, carbides and nitrides in the early VEC region (that is VEC \( \approx 8 \)) exhibit more directional covalent bonding associated with N 2p orbitals, while increasing VEC (up to 17) moves the Fermi level up to near the top of the metal d bands such that non-directional metallic bonding dominates the overall atomic bonding, leading to an increased ductility. In addition, comparing \( k \) and \( \nu \) for nitrides and carbides with the same VEC in the range of 8–11 suggests that carbides are slightly more brittle than nitrides, which may be attributed to the stronger covalent bonding character for the carbides.

The binary nitrides and carbides that are predicted to be brittle, all have a VEC of 8 or 9, while the compounds that are in the ductile region have a VEC of 10–17. In fact, the data from the left top to the right bottom approximately corresponds to an increasing VEC, as indicated by the arrow in Fig. 1(a). This implies that increasing the valence electron concentration results in improved ductility, which is in good agreement with previous reports \[17,38,55\]. In addition, the data indicates a brittle-to-ductile transition, which occurs for both carbides and nitrides at the same VEC \( \approx 10 \). This suggests that the VEC is a key parameter for classifying the mechanical properties of rocksalt structure carbides and nitrides. Correspondingly, we directly plot in Fig. 1(b) and (c) the Pugh’s and Poisson’s ratios as a function of VEC.

Pugh’s ratio \( k \), plotted in Fig. 1(b) as a function of VEC, decreases steeply when moving to the right in the periodic table, from an average of 0.64 for group 4 nitrides at VEC \( \approx 9 \) to \( k = 0.14 \) for CrN at VEC \( = 11 \), and similarly from an average of 0.74 for group 4 carbides at VEC \( = 8 \) to \( k = 0.16 \) for group 8 carbides at VEC \( = 12 \). Pugh’s ratio does not have a valid value for group 6–9 nitrides (with the exception of CrN and CoN) and for group 7–9 carbides (with the exception of MnC, FeC, RuC and CoC), because the calculated \( C_{44} \) values for these compounds are negative. For VEC \( > 12 \), Pugh’s ratio remains approximately constant and reaches an average \( k = 0.28 \) and 0.21 for group 12 nitrides and carbides with VEC = 17 and 16, respectively. Poisson’s ratio \( \nu \), as plotted in Fig. 1(c), exhibits a similar but inverse dependency on VEC. More specifically, \( \nu \)
 increases steeply from an average of 0.20 for group 4 carbides at VEC = 8 to \( r = 0.44 \) for VEC = 12, and remains relatively constant for VEC >12 to reach 0.38 and 0.41 for group 12 nitrides and carbides at VEC = 17 and 16, respectively. Both Pugh’s and Poisson’s ratios indicate a brittle-to-ductile transition at a VEC = 10. This is in good agreement with previous reports of nitrides which suggest a brittle character and cracking at failure for group 4 and 5 nitrides (VEC <10) [35–37] while carbonitrides with VEC >10 exhibit comparatively more ductility [35–37]. In addition, we note that ternary carbonitrides (pseudobinary alloys) exhibit more ductility than the binary carbides/nitrides at the same VEC. This can be attributed to the fact that in a ternary alloy, the bonds are dissimilar resulting in less symmetric bond relaxation and thus more plastic deformation [36,38,39,55].

An alternative indicator for the ductility of a material is the Cauchy pressure \( p_c = C_{12} - C_{44} \) [75], where \( p_c < 0 \) typically indicates a brittle material and correspondingly \( p_c > 0 \) classifies a material as ductile. The calculated \( p_c \) values of the binary carbides and nitrides are provided in the supplementary document. They are positive for all mechanically stable nitrides and carbides from groups 6–12, but are negative for carbides of metals from groups 4 and 5. Thus, the calculated \( p_c \) values suggest that all nitrides are ductile, while carbides with VEC ≤9 are brittle and carbides with VEC ≥10 are ductile. The latter result on carbides is in perfect agreement with the above analysis using \( r \) and \( k \). We note, that nitrides with VEC = 9 have positive but relatively small Cauchy pressures of 5, 38, and 43 GPa, suggesting that they are “ductile” but relatively close to brittle. That is, they are more ductile than the carbides with the same VEC, which is attributed to the larger radius of nitrogen vs carbon, and a correspondingly smaller covalent bonding character in the nitrides. This is also consistent with the larger average \( r = 0.24 \) for nitrides with VEC = 9 than \( r = 0.20 \) for carbides with VEC = 9. We note that another parameter that indicates a materials ductility has recently been introduced: The cooperation parameter \( \delta \). It is primarily applied to characterize the ductility of metallic glasses [76] and classifies materials as ductile if \( \delta > 1 \), corresponding to a value of \( r > 0.31 \). However, the parametric values of \( k < 0.5 \) and \( r > 0.28 \) are more commonly used for transition metal nitrides and carbides and are therefore used in this article.

**Fig. 2** is a plot of the calculated single crystal shear modulus \( C_{44} \) for group 4 to 12 transition metal nitrides and carbides as a function of valence electron concentration. The \( C_{44} \) of the nitrides decreases steeply from an average of 136 GPa for group 4 nitrides (VEC = 9) to −148 GPa for group 7 nitrides (VEC = 12) and increases again to averages of −39, 54, 39 and 46 GPa for group 9, 10, 11 and 12 nitrides (VEC = 14–17). In addition, for each fixed VEC corresponding to a specific column in the periodic table, \( C_{44} \) decreases when moving down from row IV to V to VI. The nitrides with VEC = 11–14 have negative \( C_{44} \) values (with the exception of CrN and CoN with small \( C_{44} = 8 \) and 55 GPa, respectively, which may be an artifact of our calculations that neglect Coulomb repulsion, as discussed later). The negative \( C_{44} \) indicates mechanical instability of the rock-salt structure for these nitrides. The carbides show a similar trend as the nitrides, with \( C_{44} \) decreasing for the early transition metals from an average of 164 GPa for group 4 carbides to −9 GPa for group 7 carbides (VEC = 11) and to a minimum of −47 GPa at VEC = 13, followed by an increase to 47 and 22 GPa for group 10 and 12 carbides. Negative \( C_{44} \) values for VEC = 11–13 indicate a region of mechanical instability. However, all row-four carbides, including MnC, FeC, and CoC, as well as RuC from row V with VEC = 12, exhibit positive \( C_{44} \) which may be attributed to the localization of elections as discussed below. We note that PtC and AuC with VEC = 13 and 14 have positive \( C_{44} = 19 \) and 36 GPa values, despite that they belong to groups 9 and 10. However, they are nevertheless mechanically unstable because their \( C_1 < C_{12} \). It is also imperative to mention here that some nitrides (such as CrN [24,77–80], MnN [81] and FeN [82]) and carbides such as (CrC [83] and MnC [84]) exhibit magnetic character. Of these magnetic compounds, CrN is the most important as it falls in the valence electron region of interest—that is near mechanical instability. CrN at 0 K is most stable in the antiferromagnetic orthorhombic structure [24,77,79,80,85–87], but attains a paramagnetic rocksalt structure near room temperature [77,79]. For the sake of analyzing the trends in mechanical properties, all nitrides and carbides in this investigation are considered non-magnetic and in the rocksalt structure unless indicated otherwise.

We briefly discuss here the mechanical stability/instability as evidenced by positive/negative \( C_{44} \) values of rocksalt phases in **Fig. 2**. Both nitrides and carbides exhibit a transition to mechanical instability with increasing valence electron concentration at approximately VEC = 11, and a return to stability around VEC = 14. The instability range is slightly smaller for the carbides than the nitrides, and the \( C_{44} \) values are less negative. This can be attributed to the higher covalency of metal-carbon bonds which are facilitated by the hybridization of metal \( d-\epsilon_g \) and non-metal \( p \) orbitals, resulting in a stronger resistance to shear which leads to higher \( C_{44} \) values for the carbides than the corresponding nitrides, as plotted in **Fig. 2**. The decrease in \( C_{44} \) and the onset of mechanical instability at VEC = 11 can be explained by the overlap of \( d-\epsilon_g \) orbitals which increases upon shear, resulting in an energy reduction during shearing of compounds with partially filled \( d-\epsilon_g \) orbitals [41,43,44,88]. Correspondingly, we propose that the reason for returning back to mechanical stability for group 10–12 nitrides and carbides is due to both bonding and anti-bonding \( d-\epsilon_g \) orbitals being nearly full, such that the additional overlap of \( d-\epsilon_g \) orbitals which increases the split between these orbitals has a negligible energy benefit upon shearing. This can be more quantitatively understood by analyzing the partial electron occupancies of the \( d-\epsilon_g \) orbitals and \( d-\epsilon_g \) orbitals, which are calculated by projecting the wavefunction onto local orbitals as described in Section II. The fraction of 5d-electrons that occupy \( d-\epsilon_g \) orbitals for the nitrides from group 4–12 and period VI varies as 0.62, 0.66, 0.68, 0.69 0.71, 0.69, 0.68, 0.63 and 0.61 for HN, TaN, WN, ReN, OsN, IrN, PtN, AuN and HgN, respectively, indicating a monotonic increase from 0.62 for HN
Calculated and an increasing ductility and toughness for VEC increasing above the transition metal carbonitrides and ternary nitrides. The symmetric single crystal shear modulus \( C_{44} \) as a function of van der Waals (group 4) to 0.71 for OsN (group 8) followed by a monotonic decrease to 0.61 for HgN (group 12). This trend is reflected in the plotted \( C_{44} \) for OsN of group 8, at VEC of 12, and \( C_{44} = -239 \) GPa for OsN of group 8, at VEC of 13, which has the highest fraction of \( d \) electrons occupying \( d-\epsilon_g \) orbitals.

Fig. 3 is a plot of the average single crystal shear modulus \( C_{44} = (C_{44} + C_{55} + C_{66})/3 \) for carbonitrides and ternary nitrides using group 4 to 6 transition metals. These calculations are done to explore in more detail the dependence of \( C_{44} \) on VEC for smaller than 11, and to quantify the critical VEC for the transition to mechanical instability. The plot includes carbonitrides from all nine group 4 to 6 transition metals which are formed using a rock-salt type structure with, however, a mixture of carbon or nitrogen atoms occupying the four anion sites of a conventional unit cell, such that 0.25, 50, 75, or 100% of anions are carbon and the remainder nitrogen. Similarly, the six sets of ternary metal alloy nitrides presented in Fig. 3 are all those that contain two transition metals from the same period and adjacent groups, that is group 4 and 5 or group 5 and 6. All plotted data sets contribute to an overall well defined trend of \( C_{44} \) vs VEC, indicating an approximately parabolic curve. The average \( C_{44} \) of carbonitrides of group 4 increases slightly from an average of 164 GPa for group 4 carbides at VEC = 8 to 178 GPa at VEC = 8.25, but then decreases steeply to 134 GPa for group 4 nitrides at VEC = 9. Thus, \( C_{44} \) reaches a maximum at VEC = 8.25, in agreement with previous reports which indicate a maximum strength of TiC of 134 GPa for group 4 nitrides at VEC = 8 [44,45], and an increasing ductility and toughness for VEC increasing above 8.4 for transition metal nitrides and carbonitrides [35,38,55]. The calculated \( C_{44} \) of the carbonitrides and ternary nitrides decreases to an average of 45 GPa for the ternary carbonitrides and nitrides of metallic alloys at VEC = 10.5 and an average of \( C_{44} = -39 \) GPa for group 6 nitrides at VEC = 11. This negative value indicates mechanical instability at VEC = 11. Linear interpolation of the \( C_{44} \) data sets of carbonitrides and ternary nitrides in the VEC = 10–11 region indicates that \( C_{44} = 0 \) for VEC = 10.74 for MoC\(_1\),N\(_{5}\), 10.72 for WC\(_1\),N\(_{5}\), 10.48 for NbC\(_1\), 10.46 for Ta\(_{1}\),W\(_{4}\),N. Correspondingly, the average critical VEC that leads to mechanical instability is 10.6 ± 0.2. All calculated carbonitrides and ternary nitrides of Cr with VEC = 10–11 have positive \( C_{44} \) values and are not considered here for the calculation of the critical VEC for mechanical instability. The trend of a decreasing \( C_{44} \) with increasing VEC is consistent with previous theoretical reports which report a similar decrease in \( C_{44} \) with increasing VEC [89–93].

Special quasi-random structure calculations of two specific ternaries, Ta\(_{0.25}\)W\(_{0.75}\)N and WC\(_{0.25}\)N\(_{0.75}\), yield \( C_{44} = -10 \) and -6 GPa, respectively, in good agreement with previous studies which report a \( C_{44} \) of -15 GPa for Ta\(_{0.25}\)W\(_{0.75}\)N [90]. These two compositions both have a VEC = 10.75 and are chosen because they are near the mechanical stability-instability transition. The data points are also plotted as black open symbols in Fig. 3 at VEC = 10.75. The values are slightly higher (by 12 and 9 GPa) than the \( C_{44} \) = -22 and -15 GPa obtained using the 8 atom unit cells for the same compositions. This suggests that disorder has only a minor effect on the calculated VEC for transition to mechanical instability. The approximately 10 GPa higher values from the SQS calculations indicate that random occupation on cation or anion sites mechanically stabilizes the rocksalt structure. This leads to a slight increase in the expected critical VEC for the instability-instability transition. More quantitatively, using the average slope \( \delta C_{44}/\delta \text{VEC} = 183 \) GPa we estimate a correction of -0.05. However, this correction is smaller than the above stated uncertainty, such that we retain a value of VEC = 10.6 ± 0.2 for the instability-instability transition.

Fig. 4(a) shows the bulk modulus \( B \) as a function of VEC for the same carbonitrides and ternary nitrides discussed above, using group 4 to 6 transition metals. The plot indicates that the different ternary nitrides and carbonitrides with the same VEC exhibit similar values, typically within a range of ~50 GPa. \( B \) increases approximately linearly from an average of 255 GPa for VEC = 8 to \( B = 314 \) and 360 GPa for VEC = 9 and 10, but remains approximately constant for VEC > 10, with an average of 366 GPa for VEC = 11. These values are in reasonable agreement with previously published bulk moduli of binary carbides and nitrides, with predicted averages of 234, 286, 330 and 334 GPa for VEC = 8, 9, 10 and 11, respectively [94].

The plot of the tensile elastic constant \( C_{11} = (C_{11} + C_{22} + C_{33})/3 \) vs VEC in Fig. 4(b) indicates an initial steep increase from an average of 499 GPa for group 4 carbides with VEC = 8 to 579 GPa for group 4 nitrides with VEC = 9. However, the data for VEC > 9 shows large variations in \( C_{11} \) for different nitrides and carbonitrides with the same VEC, leading to the rather noisy data in Fig. 4(b) and no evident trend of \( C_{11} \) vs VEC > 9. That is, in contrast to \( C_{44} \) which is a strong function of the VEC as shown in Fig. 3, the VEC is a less useful parameter in determining \( C_{11} \) of the investigated nitrides and carbonitrides. This is consistent with previous reports which indicate an increase in the tensile elastic constant \( C_{11} \) with increasing VEC from 8 to 10, and a subsequent decrease to group 6 nitrides with VEC = 11 [89–93]. We note here that elastic constants of the ternary nitrides, carbides and carbonitrides can also be strongly affected by atomic ordering. Some carbides and nitrides have been reported to exhibit ordering on the metal sublattice [95–97] to form more thermodynamically stable structures. Vacancy ordering on the non-metal sublattice has also been observed for some carbides [98,99]. Both short range or long range ordering has the potential to affect the elastic constants. However, a systematic study of ordering effects is beyond the scope of this study, since it not only requires large-cell SQS calculations but also relaxation of the cubic symmetry into less symmetric e.g. rhombohedral ordered structures.

Fig. 5(a) is a plot of the isotropic elastic modulus \( E \) of carbonitrides and ternary nitrides vs VEC, as determined from the elastic constants presented in Figs. 3 and 4. \( E \) increases slightly from an
average 426 GPa for group 4 carbides at VEC = 8 to $E = 469$ GPa for VEC = 9, followed by a steep decrease to an average 234 GPa for VEC = 10.5 and 83 GPa for VEC = 11. The calculated elastic moduli are in excellent agreement (4–20% deviation) with the measured elastic moduli for early transition metal carbides with VEC = 8–10 [21–23,100–102] but considerably (27–78%) deviate from the measured moduli for carbonitrides with VEC = 10–11 [26,36,37,103–106]. We attribute the deviations primarily to microstructural effects associated with residual stress in thin films as well as variations in composition, density and grain size. In addition, the presence of vacancies can significantly alter the mechanical properties for carbonitrides [100], most prominently if they are near mechanical instability such as group 6 nitrides [36,41,42,107]. Despite some deviations, the cited experimental reports confirm the calculated trend of decreasing elastic moduli with increasing VEC which is attributed to the increasing occupation of $d$–$t_{2g}$ orbitals as mentioned previously.

Fig. 5(b) is a plot of the Hardness $H$ vs the VEC for carbonitrides and ternary nitrides, as determined from the calculated elastic tensor using Tian’s model [73]. The data indicates that the hardness is similar (±3 GPa) for different compounds that have the same VEC, but that $H$ decreases with increasing VEC, showing a similar trend as $C_{44}$ and $E$. More specifically, the average $H$ decreases steeply from 25 GPa for group 4 carbides with VEC = 8 to $H = 4$ GPa at VEC = 10.5. For VEC > 10.5, the carbonitrides MoC$_{1-x}$N$_x$ and WC$_{1-x}$N$_x$ and the ternary nitrides Nb$_{1-x}$Mo$_x$N and Ta$_{1-x}$W$_x$N have no valid hardness because of their negative $C_{44}$, while $H$ of V$_{1-x}$Cr$_x$N and CrC$_{1-x}$N$_x$ continues to decrease and reaches 2 GPa at VEC = 11. Comparing our hardness predictions in Fig. 5(b) with reported experimental nanoindentation data from epitaxial layers suggests that the agreement is relatively good (<25% deviation) for group 4 nitrides [21,100,101], but that the calculated hardness differs considerably (49–87% deviation) from experimentally measured values for binary nitrides, carbides and carbonitrides with VEC > 10 [23,37,103,106,108]. We attribute the deviations to residual stress, composition variations, vacancy concentrations, and crystalline defects in the experimental specimens, which are expected to be more dominant at higher VEC where the rocksalt structure may only be metastable and/or vacancies are energetically stable.

The dynamical stability of the binary nitrides and carbides of group 4–7 transition metals are explored by calculating the phonon dispersion curves as described in the procedure section. This is done because both dynamical and mechanical stability is necessary to determine the overall stability of the material. Imaginary frequencies near the Brillouin zone boundaries indicate dynamical instability, which may occur in some compounds despite that they are stable based on the static elastic stability criteria [109–111]. The nitrides of group 4 transition metals exhibit only real frequencies, indicating dynamical stability. In contrast, those for groups 5 and 6...
also show some imaginary frequencies, that is, nitrides are unstable for VEC > 10. That means, nitrides of group 5 are dynamically unstable despite that they are elastically stable. Similarly, the carbides are dynamically stable for groups 4 and 5 but unstable for group 6, corresponding to a dynamic instability for VEC > 10, although they are elastically stable for group 6. In other words, a VEC of 10 is sufficient to cause dynamical instability in both binary carbides and nitrides while, in contrast, VEC > 10 is required for elastic instability. A detailed quantitative determination of the critical VEC for dynamical instability is beyond the scope of the present work and is therefore not presented in this paper. We reiterate here that the mechanical stability of nitrides and carbides beyond VEC = 11 does not necessarily guarantee dynamical stability and that a systematic investigation of the phonon dispersion curves for mechanically stable binary nitrides and carbides with VEC > 11 would be required.

Fig. 6 shows representative phonon dispersion curves from three nitrides, HfN, TaN and WN, corresponding to valence electron concentrations of 9, 10 and 11, respectively. The calculated phonon frequency f is plotted along high symmetry directions $\Gamma - X - W - K - \Gamma$ in the Brillouin zone, where frequencies below the dotted horizontal line $f = 0$ correspond to imaginary frequencies, indicating that a static displacement of atoms with the corresponding wavelength leads to an energy reduction. The phonon frequency of HfN is real throughout the Brillouin zone, with $f = 0$–5 and 12–17 THz for the acoustic and optical branches, respectively, as shown by the navy-blue curves in Fig. 6. The longitudinal acoustic branch of the HfN dispersion curves shows anomalies along all three plotted high-symmetry $\Gamma - X$, $\Gamma - K$ and $\Gamma - L$ directions, corresponding to [100], [110] and [111] crystalline axes. For example, the branch along $\Gamma - X$ exhibits a softening near $q = 0.7$. Such softening has previously been reported for TiN [94,112], ZrN [94,113] and HfN [94,112,113] and has been attributed to the nesting of the Fermi surface in group 4 transition metal nitrides [114], leading to strong electron phonon coupling and a relatively high critical temperature for superconductivity [112,115]. Group 5 carbides have the same VEC = 9 as the group 4 nitrides and exhibit similar phonon spectra [94]. Hence, rocksalt structure nitrides and carbides with VEC = 9 exhibit fully real phonon spectra and are therefore dynamically stable up to their melting points. The olive-green curve in Fig. 6 shows the vibrational frequencies for TaN, which has one valence electron more than HfN and belongs to the group 5 nitrides with VEC = 10. The plot indicates the acoustic and optical branches to range from 6 to 3 THz and from 6 to 17 THz, respectively. The TaN optical modes extend over a wider frequency range than for HfN, resulting in some relatively soft optical phonon modes for TaN at approximately the middle between the zone center and boundary. More importantly, both the longitudinal and transversal acoustic branches exhibit imaginary frequencies at and near the X and K points of the Brillouin zone, including the path along the boundary from X to W to K. This is consistent with previous reports of the phonon spectra for the group 5 nitrides [94,114,116–118], and is attributed to the relatively high d-2g electron density near the Fermi level. Thus, displacement of neighboring cations in opposite directions promotes increased d–2g orbital overlap and a corresponding energy reduction which results in dynamical instability [94,114,117,118]. The other nitrides and carbides with VEC = 10, namely group 5 nitrides and group 6 carbides, have qualitatively similar phonon dispersion curves as TaN. That is, they exhibit some imaginary frequencies and are therefore dynamically unstable despite that they are mechanically stable based on the calculated elastic tensor. However, we note here that finite temperature effects can stabilize these dynamically unstable branches as reported for VN [22,116].

Fig. 6 also shows the phonon dispersion curves for WN, representing the group 6 nitrides with VEC = 11, represented by the red curve. The optical phonon frequencies range from 9 to 18 THz and the acoustic phonons have a maximum frequency of 5 THz and a plotted minimum of –10 THz, indicating imaginary frequencies and dynamical instability, consistent with previously reported phonon dispersion curves of group 6 nitrides [94,113]. In contrast to TaN, the transverse acoustic branch for WN is imaginary for almost the entire Brillouin zone, indicating instability for phonons travelling along [100] and [110] directions for nearly all wavelengths. This is attributed to the instability of WN against shear along [100], as evident from the negative $C_{44}$ presented in Figs. 2 and 3. This instability is due to the increasing overlap of d-2g orbitals which stabilize sheared structures with a reduced cation–cation distance along some of the [110] crystalline directions [41]. Nitrides and carbides with VEC = 11 could become dynamically stable at high temperatures. A systematic study needs to be done to verify this. We also note here that the elastic constants calculated at 0 K are still valid at finite temperatures because the phonon branches near the $\Gamma$ point are not affected by temperature, as previously studied in detail for VN at various temperatures [116].

**Effect of magnetization, Hubbard parameter, vacancies and temperature on the elastic constants and on the mechanical stability-instability transition:**

There are various important parameters that have not been included in the above quantitative presentation of the mechanical properties of transition metal nitrides and carbides vs VEC. They have been neglected for consistency and simplicity purposes, but are discussed in the following, including the effect of the magnetization, the Hubbard parameter $U$, vacancies and the temperature. In order to explore the effect of magnetic ordering in nitrides and carbides containing Cr, Mn, Fe, Co, or Ni, spin-polarized calculations are performed to compare ferromagnetic and anti-ferromagnetic ordering with the non-magnetic structures. For these calculations, atomic positions are relaxed while forcing the overall structure to remain cubic, as explained in the procedure section. For the anti-ferromagnetic (AFM) structures, ordering along [100], [111] and [110] is considered. $C_{44}$ calculations are performed on the lowest energy structure of each compound. Both CrN and MnN are found to be structurally stable in the [110] AFM configuration with $C_{44}$ values of 126 and 122 GPa and with magnetic moments of 2.99
and 2.19 µg/cation, respectively, while their carbides are found to be stable in ferromagnetic configurations resulting in a $C_{44}$ of 89 and 100 GPa with magnetic moments of 0.97 and 1.35 µg/cation, respectively. FeN is most stable in the ferromagnetic phase yielding $C_{44} = 69$ GPa with a magnetic moment of 2.29 µg/cation. CoN, CoC, NiN and NiC are non-magnetic. In summary, the spin-polarized calculations indicate that CrN, MnN, FeN are magnetically stabilized. The quantitative results are also included in the table provided as supplementary document.

The Hubbard parameter $U$ can be used within DFT to account for on-site Coulomb repulsion, which can affect the mechanical properties due to the relatively localized $d$ electrons in transition metal nitrides and carbides. For example, $U = 3$ eV for CrN has been reported to yield a $C_{44} = -116$ GPa [79,87], while the non-magnetic CrN with $U = 0$ in our study exhibits a positive $C_{44} = 8$ GPa. That is, Coulomb repulsion mechanically destabilizes CrN. Similarly, we expect Coulomb repulsion to reduce the mechanical stability of other compounds and envision that, as a consequence, all cubic nitrides and carbides with VEC = 11–14 are mechanically unstable, despite that some of them (CrN, CoN, MnC, FeC, RuC, and CoC) are reported to yield a $C_{44} = 69$ GPa with a magnetic moment of 2.29 µg/cation. CoN, CoC, NiN and NiC are non-magnetic. In summary, the spin-polarized calculations indicate that CrN, MnN, FeN are magnetically stabilized. The quantitative results are also included in the table provided as supplementary document.

Temperature is another parameter that affects the elastic constants and possibly also the critical VEC value for the stability-instability transition. Previous studies on the effect of temperature on the elastic constants $C_{11}$, $C_{12}$ and $B$ indicate that increasing temperature typically causes an increase in the lattice constants and a corresponding decrease in the bulk modulus and $C_{11}$ and $C_{12}$ [119–125]. This can be attributed to bond softening associated with the anharmonicity of the vibrational modes of atoms. In contrast, the relationship between $C_{44}$ and the temperature is not as simple: For mechanically stable systems with a positive $C_{44}$, temperature is expected to decrease $C_{44}$, just like $C_{11}$ and $B$ [119–125]. However, for systems with a negative $C_{44}$, increasing temperature could lead to augmentation of $C_{44}$ because of the concave energy surface along directions of instability [126–128]. Thus, the effect of temperature on $C_{44}$ is convoluted and high temperature calculations would need to be performed to accurately determine its effect. Nevertheless, we expect nitrides, carbides and carbonitrides with a VEC <11 to exhibit a $C_{44}$ which decreases with increasing temperature while the temperature effect for the mechanically unstable structures remains uncertain.

Therefore, temperature may stabilize nitrides and carbides that exhibit a dynamical instability at 0 K. Such stabilization by temperature has previously been reported for the case of VN [116] and may also affect mechanical instability. Correspondingly, we expect temperature to reduce the magnitude of the slope in the $C_{44}$ vs VEC plots in Figs. 2 and 3 by reducing $C_{44}$ of the early transition metal carbonitrides and increasing $C_{44}$ for the mechanically unstable structures with VEC = 11–14. This will also reduce the slopes of the $E$ vs VEC and $H$ vs VEC plots in Fig. 5, as $E$ and $H$ are strongly affected by changes in $C_{44}$, as discussed above. Consequently, the critical VEC for mechanical instability is expected to increase with increasing temperature.

Vacancies on both cation and anion sublattices are common for most experimentally synthesized transition metal carbides [83,99,129,130] and nitrides [95,97,100,131–134] and are known to influence the crystal structure [25,26,107,135] as well as the electrical [136–138] and mechanical properties [36,44,100,139]. Vacancies in early rocksalt structure carbonitrides including HN, and TiN, cause a softening of the elastic constant matrix and the elastic modulus [100,140,141]. However, for carbonitrides of group 6, vacancies are predicted to augment elastic constants and stabilize the rocksalt structure [25,41,142]. Thus, similar to temperature, vacancies are expected to reduce the slope of the elastic constants vs VEC plots, leading to an increase the critical VEC at which mechanical instability occurs. We note that this argument assumes that the VEC here is defined by the average number of electrons of the cations plus anions, neglecting the fact that vacancies themselves reduce the average number of electrons and therefore the VEC. This reduction in VEC through vacancies may in fact contribute to the stabilization of the rocksalt phase of e.g. group 6 nitrides, since a reduction in VEC leads to an increase in $C_{44}$, as shown in Figs. 2 and 3.

4. Conclusions

Calculated mechanical properties ($\nu$, $k$, $B$, $T_{11}$, $T_{44}$, $E$, $H$) of rocksalt structure transition metal nitrides, carbonitrides and ternary nitrides indicate a steady increase in ductility with increasing VEC, with a brittleness-to-ductile transition at $T_{11} = 10$. The single crystal shear modulus $T_{44}$ decreases from an average of 161 GPa for group 4 carbides to 39 GPa for group 6 nitrides, indicating a transition to mechanical instability at a critical valence electron concentration VEC = 10.6. In contrast, $T_{11}$ increases from an average of 499 GPa for group 4 carbides to 681 and 614 GPa for group 5 and group 6 nitrides, while $B$ increases over the same VEC = 8–11 range from an average of $B = 255–366$ GPa. The isotropic elastic modulus and hardness decrease steeply with increasing VEC. This is primarily attributed to the decreasing $C_{44}$ which is caused by the increasing electron density in d- and p-orbitals, resulting in an energy reduction upon shear and a corresponding softening with increasing VEC. Calculated phonon dispersion curves indicate a dynamical stability-instability transition between $T_{11} = 9$ and 10, which is below the critical VEC for mechanical instability of 10.6. Additionally, the mechanical stability is also affected by (i) magnetic ordering which stabilizes cubic CrN, MnN, FeN, (ii) on-site Coulomb repulsion which is accounted for by the Hubbard parameter and reduces the stability of compounds with an increasing VEC such that a considerable fraction of the d bands is filled, and (iii) temperature and point defects which both reduce $T_{44} = 0$ and increase $C_{44} < 0$, possibly causing an increase in the critical VEC for the stability-instability transition.

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Appendix A. Supplementary data

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structural, mechanical and electronic properties of 


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