

Review of complex numbers

$$i = \sqrt{-1} \rightarrow i^2 = -1 \quad \text{"imaginary number"}$$

$$\sqrt{-9} = 3i$$

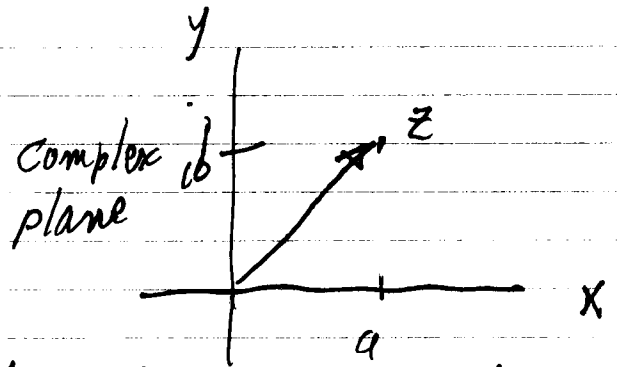
In general $z = a + bi$ ← complex number
 ↑ ↑
 real imaginary
 part part

$$a = \operatorname{Re} z$$

$$b = \operatorname{Im} z$$

$$|z| = \sqrt{a^2 + b^2} = \text{modulus of } z$$

$$|z|^2 = a^2 + b^2 = \text{length of complex vector in complex plane}$$

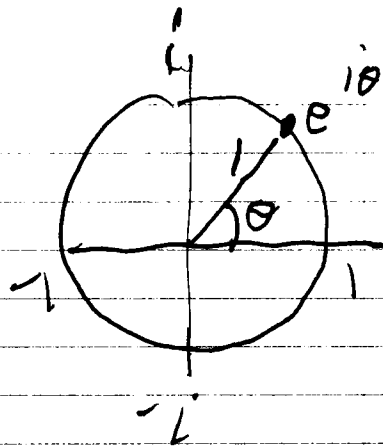


Multiplication of complex numbers

$$z_1 z_2 = (a + ib)(c + id) = (ac - bd) + i(bt + ad)$$

$$z_1 + z_2 = (a + c) + i(b + d)$$

De Moivre's Theorem



$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$|e^{i\theta}| = \sqrt{\cos^2 \theta + \sin^2 \theta} = 1$$

$$z_1 = r_1 e^{i\theta_1} = r_1 \cos \theta_1 + i r_1 \sin \theta_1$$

$$z_2 = r_2 e^{i\theta_2} = r_2 \cos \theta_2 + i r_2 \sin \theta_2$$

$$z_1 z_2 = (r_1 r_2) e^{i(\theta_1 + \theta_2)}$$

$$|z_1 z_2| = |z_1| |z_2| = r_1 r_2$$