Physics 6320/7320

Problem set #1. Due W 2 September 2009

Show your work and explain clearly.

1 Are the following sets of functions linearly independent or dependent?

(a) $\cos x\,,\ e^{ix}\,,\ 3\sin x$

(b) $7\,,\ x^2\,,\ 9x^4\,,\ e^{-x}$

(c) $x\,,\ x+2\,,\ x+5$

(d) $x\,,\ (x-1)^2\,,\ (x+1)^2$

2 **A**t time $t = 0$ the state of a certain system is given as

$$|\,\psi(0)\,\rangle \;=\; \sqrt{\frac{1}{3}}\,|\,\phi_1\,\rangle \;+\; \sqrt{\frac{1}{4}}\,|\,\phi_2\,\rangle \;+\; \sqrt{\frac{1}{6}}\,|\,\phi_3\,\rangle \;+\; \sqrt{\frac{1}{4}}\,|\,\phi_4\,\rangle$$

The four basis vectors $|\,\phi_n\,\rangle$ are energy eigenstates with eigenvalues $E_1\,,\ E_2\,,\ E_3\,,\ E_4$ respectively.

(a) What are the possible results of measuring the energy of this system, and with what probabilities will they occur?

(b) At a later time t, what is the corresponding expansion of $|\,\psi(t)\,\rangle$ in this same basis?

(c) Find the expectation value of the Hamiltonian at time $t = 0$ and at a later time $t > 0$.

3 **C**onsider a system whose Hamiltonian and initial state are

$$H \;=\; \mathcal{E}_0 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad |\,\psi_0\,\rangle \;=\; \sqrt{\frac{1}{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\,.$$

(a) Find the eigenvalues and eigenvectors of H.

(b) What values will we obtain when measuring the energy, and with what probabilities?

(c) Calculate $\langle\, H\,\rangle$, the expectation value of the Hamiltonian.

(d) Compare the results of (b) and (c); explain.

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4 Consider a system whose state is

$$|\,\psi\,\rangle \;=\; \sqrt{\frac{1}{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}.$$

Two observables of this system are A and B, given by the matrix representations

$$A \;=\; \frac{1}{2}\begin{pmatrix}0&0&0\\0&3&-1\\0&-1&3\end{pmatrix} \quad\text{and}\quad B \;=\; \frac{1}{2}\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&-2\end{pmatrix}.$$

(a) With the system prepared in state $|\,\psi\,\rangle$ we perform a measurement where A is measured first, and then B immediately afterwards. Find the probability of obtaining the values $A=0\,,\ B=0\,.$

(b) Again we prepare the system in state $|\,\psi\,\rangle$ but now we measure B first, and then A immediately afterwards. Find the probability of obtaining the values $A=0\,,\ B=0$ in this case.

(c) Compare the results of (a) and (b). Explain.

(d) Repeat for getting the results $A=1\,,\ B=1$ in state $|\,\psi\,\rangle$, again noticing how the results depend on the order of the measurements.

(e) Are the observables A and B ***compatible***? Explain.

5 **Time dependence of an observable in a closed system.**

Given a system with hamiltonian $\mathbf{H}$ and an observable $\mathbf{A}$:

$$\mathbf{H} = \hbar\omega\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix}a_1&0\\0&a_2\end{pmatrix}$$

(a) Is $\mathbf{A}$ conserved? (Check the commutator.)

(b) Find the eigenvalues λ_1, λ_2 and orthonormal eigenvectors $|\,\phi_1\,\rangle\,,\,|\,\phi_2\,\rangle$ of $\mathbf{H}$.

(c) Given the initial state $|\,\psi(0)\,\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, find $|\,\psi(t)\,\rangle$ for $t>0$.. Write your answer both in form $|\,\psi(t)\,\rangle = f_1(t)\,|\,\phi_1\,\rangle + f_2(t)\,|\,\phi_2\,\rangle$, and in form $|\,\psi(t)\,\rangle = \begin{pmatrix}\alpha(t)\\ \beta(t)\end{pmatrix}$.

(d) In the initial state, what are the possible results of measuring $\mathbf{A}$, and their corresponding probabilities P_1, P_2?

(e) In the state $|\,\psi(t)\,\rangle$, what are the possible results of measuring $\mathbf{A}$, and their corresponding probabilities $P_1(t), P_2(t)$? Is the total probability of some result conserved?

(f) ***Sketch a graph*** showing the functions $P_1(t), P_2(t)$ on the same clearly-labelled axes.