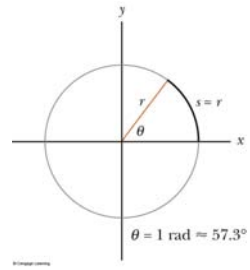
Chapter 7

Rotational Motion
and
The Law of Gravity

The Radian

- The radian is a unit of angular measure
- The radian can be defined as the arc length s along a circle divided by the radius r
- $\theta = \frac{s}{r}$



More About Radians

- Comparing degrees and radians

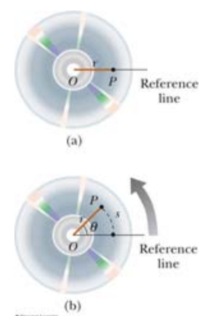
$$1\,\text{rad} = \frac{360°}{2\pi} = 57.3°$$

- Converting from degrees to radians

$$\theta\,[\text{rad}] = \frac{\pi}{180°}\,\theta\,[\text{degrees}]$$

Angular Displacement

- Axis of rotation is the center of the disk
- Need a fixed reference line
- During time t, the reference line moves through angle θ



Rigid Body

- Every point on the object undergoes circular motion about the point O
- All parts of the object of the body rotate through the same angle during the same time
- The object is considered to be a **rigid body**
  - This means that each part of the body is fixed in position relative to all other parts of the body

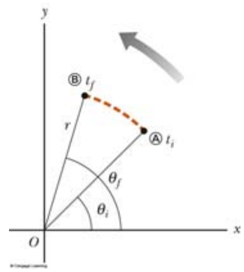
Angular Displacement, cont.

- The *angular displacement* is defined as the angle the object rotates through during some time interval
- $\Delta\theta = \theta_f - \theta_i$
- The unit of angular displacement is the radian
- Each point on the object undergoes the same angular displacement

Average Angular Speed

- The average angular speed, ω, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



Angular Speed, cont.

- The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero
- Units of angular speed are radians/sec
  - rad/s
- Speed will be positive if θ is increasing (counterclockwise)
- Speed will be negative if θ is decreasing (clockwise)

Average Angular Acceleration

- The average angular acceleration α of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Angular Acceleration, cont

- Units of angular acceleration are rad/s²
- Positive angular accelerations are in the counterclockwise direction and negative accelerations are in the clockwise direction
- When a rigid object rotates about a fixed axis, every portion of the object has the same angular speed and the same angular acceleration

Angular Acceleration, final

- The sign of the acceleration does not have to be the same as the sign of the angular speed
- The instantaneous angular acceleration is defined as the limit of the average acceleration as the time interval approaches zero

Analogies Between Linear and Rotational Motion

Linear Motion with a Constant (Variables: x and v)	Rotational Motion about a Fixed Axis with α Constant (Variables: θ and ω)	
$v = v_i + at$	$\omega = \omega_i + \alpha t$	[7.7]
$\Delta x = v_i t + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	[7.8]
$v^2 = v_i^2 + 2a\Delta x$	$\omega^2 = \omega_i^2 + 2\alpha\Delta\theta$	[7.9]

Relationship Between Angular and Linear Quantities

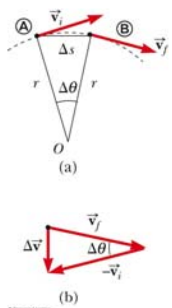
- Displacements
  $s = \theta r$
- Speeds
  $v_t = \omega r$
- Accelerations
  $a_t = \alpha r$
- Every point on the rotating object has the same angular motion
- Every point on the rotating object does *not* have the same linear motion

Centripetal Acceleration

- An object traveling in a circle, even though it moves with a constant speed, will have an acceleration
- The centripetal acceleration is due to the change in the *direction* of the velocity

Centripetal Acceleration, cont.

- Centripetal refers to "center-seeking"
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion



Centripetal Acceleration, final

- The magnitude of the centripetal acceleration is given by

$$a_c = \frac{v^2}{r}$$

  - This direction is toward the center of the circle

Centripetal Acceleration and Angular Velocity

- The angular velocity and the linear velocity are related ($v = \omega r$)
- The centripetal acceleration can also be related to the angular velocity

$$a_C = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\,\omega^2$$

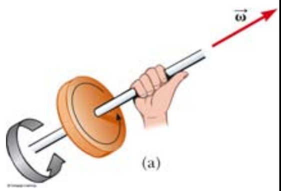
Total Acceleration

- The tangential component of the acceleration is due to changing speed
- The centripetal component of the acceleration is due to changing direction
- Total acceleration can be found from these components

$$a = \sqrt{a_t^2 + a_C^2}$$

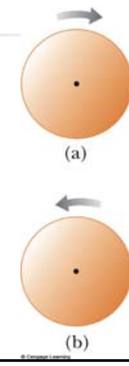
Vector Nature of Angular Quantities

- Angular displacement, velocity and acceleration are all vector quantities
- Direction can be more completely defined by using the right hand rule
  - Grasp the axis of rotation with your right hand
  - Wrap your fingers in the direction of rotation
  - Your thumb points in the direction of ω



Velocity Directions, Example

- In a, the disk rotates clockwise, the velocity is into the page
- In b, the disk rotates counterclockwise, the velocity is out of the page



Acceleration Directions

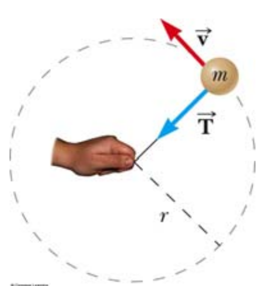
- If the angular acceleration and the angular velocity are in the same direction, the angular speed will increase with time
- If the angular acceleration and the angular velocity are in opposite directions, the angular speed will decrease with time

Forces Causing Centripetal Acceleration

- Newton’s Second Law says that the centripetal acceleration is accompanied by a force
  - $F_C = ma_C$
  - F_C stands for any force that keeps an object following a circular path
    - Tension in a string
    - Gravity
    - Force of friction

Centripetal Force Example

- A ball of mass *m* is attached to a string
- Its weight is supported by a frictionless table
- The tension in the string causes the ball to move in a circle



Centripetal Force

- General equation $F_C = m a_C = \frac{m v^2}{r}$
- If the force vanishes, the object will move in a straight line tangent to the circle of motion
- Centripetal force is a classification that includes forces acting toward a central point
  - It is *not* a force in itself

Problem Solving Strategy

- **Draw a free body diagram**, showing and labeling all the forces acting on the object(s)
- **Choose a coordinate system** that has one axis perpendicular to the circular path and the other axis tangent to the circular path
  - The normal to the plane of motion is also often needed

Problem Solving Strategy, cont.

- **Find the net force toward the center** of the circular path (this is the force that causes the centripetal acceleration, F_C)
- **Use Newton's second law**
  - The directions will be radial, normal, and tangential
  - The acceleration in the radial direction will be the centripetal acceleration
- **Solve for the unknown(s)**

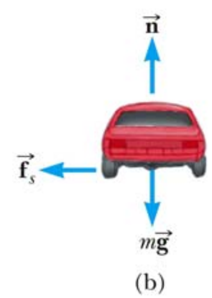
Applications of Forces Causing Centripetal Acceleration

- Many specific situations will use forces that cause centripetal acceleration
  - Level curves
  - Banked curves
  - Horizontal circles
  - Vertical circles

Level Curves

- Friction is the force that produces the centripetal acceleration
- Can find the frictional force, μ, or v

$v = \sqrt{\mu r g}$

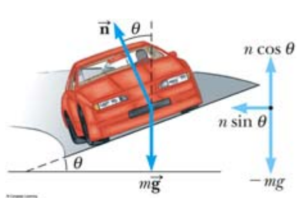


Banked Curves

- A component of the normal force adds to the frictional force to allow higher speeds

$$\tan\theta = \frac{v^2}{rg}$$

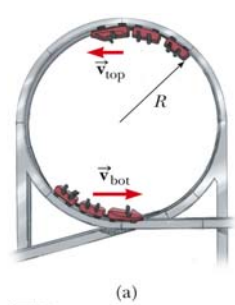
$$or\ a_c = g\tan\theta$$



Vertical Circle

- Look at the forces at the top of the circle
- The minimum speed at the top of the circle can be found

$$v_{top} = \sqrt{gR}$$



Forces in Accelerating Reference Frames

- Distinguish real forces from fictitious forces
- "Centrifugal" force is a fictitious force
- Real forces always represent interactions between objects

Newton's Law of Universal Gravitation

- If two particles with masses m_1 and m_2 are separated by a distance r, then a gravitational force acts along a line joining them, with magnitude given by

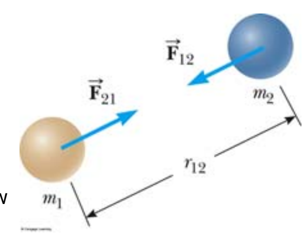
$$F = G\frac{m_1 m_2}{r^2}$$

Universal Gravitation, 2

- G is the constant of universal gravitational
- $G = 6.673 \times 10^{-11}\ \text{N m}^2/\text{kg}^2$
- This is an example of an *inverse square law*
- The gravitational force is always attractive

Universal Gravitation, 3

- The force that mass 1 exerts on mass 2 is equal and opposite to the force mass 2 exerts on mass 1
- The forces form a Newton's third law action-reaction

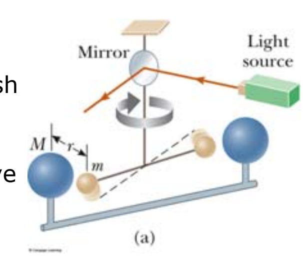


Universal Gravitation, 4

- The gravitational force exerted by a uniform sphere on a particle outside the sphere is the same as the force exerted if the entire mass of the sphere were concentrated on its center
  - This is called Gauss' Law

Gravitation Constant

- Determined experimentally
- Henry Cavendish
  - 1798
- The light beam and mirror serve to amplify the motion



Applications of Universal Gravitation

- Acceleration due to gravity
- g will vary with altitude

$$g = G\frac{M_E}{r^2}$$

TABLE 7.1

Free-Fall Acceleration *g* at Various Altitudes

Altitude (km)[a]	*g* (m/s²)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

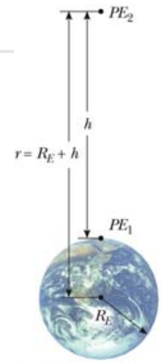
[a]All figures are distances above Earth's surface.

Gravitational Potential Energy

- PE = mgy is valid only near the earth's surface
- For objects high above the earth's surface, an alternate expression is needed

$$PE = -G\frac{M_E m}{r}$$

  - Zero reference level is infinitely far from the earth



Escape Speed

- The escape speed is the speed needed for an object to soar off into space and not return

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

- For the earth, v_{esc} is about 11.2 km/s
- Note, v is independent of the mass of the object

Various Escape Speeds

- The escape speeds for various members of the solar system
- Escape speed is one factor that determines a planet's atmosphere

TABLE 7.2

Escape Speeds for the Planets and the Moon

Planet	v_e (km/s)
Mercury	4.3
Venus	10.3
Earth	11.2
Moon	2.3
Mars	5.0
Jupiter	60.0
Saturn	36.0
Uranus	22.0
Neptune	24.0
Pluto[a]	1.1

[a]In August 2006, the International Astronomical Union adopted a definition of a planet that separates Pluto from the other eight planets. Pluto is now defined as a "dwarf planet" (like the asteroid Ceres).

Kepler's Laws

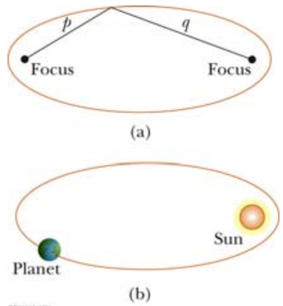
- All planets move in elliptical orbits with the Sun at one of the focal points.
- A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

Kepler's Laws, cont.

- Based on observations made by Brahe
- Newton later demonstrated that these laws were consequences of the gravitational force between any two objects together with Newton's laws of motion

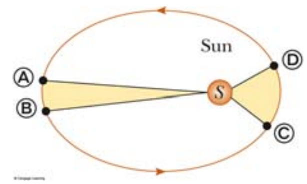
Kepler's First Law

- All planets move in elliptical orbits with the Sun at one focus.
  - Any object bound to another by an inverse square law will move in an elliptical path
  - Second focus is empty



Kepler's Second Law

- A line drawn from the Sun to any planet will sweep out equal areas in equal times
  - Area from A to B and C to D are the same



Kepler's Third Law

- The square of the orbital period of any planet is proportional to cube of the average distance from the Sun to the planet.

  $T^2 = Kr^3$

  - For orbit around the Sun, $K = K_S = 2.97 \times 10^{-19}\ s^2/m^3$
  - K is independent of the mass of the planet

Kepler's Third Law, cont

- Can be used to find the mass of the Sun or a planet
- When the period is measured in Earth years and the semi-major axis is in AU, Kepler's Third Law has a simpler form
  - $T^2 = a^3$

Communications Satellite

- A geosynchronous orbit
  - Remains above the same place on the earth
  - The period of the satellite will be 24 hr
- $r = h + R_E$
- Still independent of the mass of the satellite