

Chapter 13

Vibrations and Waves

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Hooke's Law

- $F_s = -kx$
 - F_s is the spring force
 - k is the spring constant
 - It is a measure of the stiffness of the spring
 - A large k indicates a stiff spring and a small k indicates a soft spring
 - x is the displacement of the object from its equilibrium position
 - $x = 0$ at the equilibrium position
 - The negative sign indicates that the force is always directed opposite to the displacement

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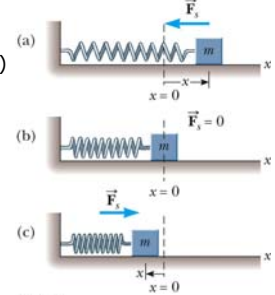
Hooke's Law Force

- The force always acts toward the equilibrium position
 - It is called the *restoring force*
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

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Hooke's Law Applied to a Spring – Mass System

- When x is positive (to the right), F is negative (to the left)
- When $x = 0$ (at equilibrium), F is 0
- When x is negative (to the left), F is positive (to the right)



Please replace with active fig. 13.1

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Motion of the Spring-Mass System

- Assume the object is initially pulled to a distance A and released from rest
- As the object moves toward the equilibrium position, F and a decrease, but v increases
- At $x = 0$, F and a are zero, but v is a maximum
- The object's momentum causes it to overshoot the equilibrium position

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Motion of the Spring-Mass System, cont

- The force and acceleration start to increase in the opposite direction and velocity decreases
- The motion momentarily comes to a stop at $x = -A$
- It then accelerates back toward the equilibrium position
- The motion continues indefinitely

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Simple Harmonic Motion

- Motion that occurs when the net force along the direction of motion obeys Hooke's Law
 - The force is proportional to the displacement and always directed toward the equilibrium position
- The motion of a spring mass system is an example of Simple Harmonic Motion

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Simple Harmonic Motion, cont.

- Not all periodic motion over the same path can be considered Simple Harmonic motion
- To be Simple Harmonic motion, the force needs to obey Hooke's Law

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Amplitude

- Amplitude, A
 - The amplitude is the maximum position of the object relative to the equilibrium position
 - In the absence of friction, an object in simple harmonic motion will oscillate between the positions $x = \pm A$

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Period and Frequency

- The period, T , is the time that it takes for the object to complete one complete cycle of motion
 - From $x = A$ to $x = -A$ and back to $x = A$
- The frequency, f , is the number of complete cycles or vibrations per unit time
 - $f = 1 / T$
 - Frequency is the reciprocal of the period

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Acceleration of an Object in Simple Harmonic Motion

- Newton's second law will relate force and acceleration
- The force is given by Hooke's Law
- $F = -kx = ma$
 - $a = -kx / m$
- The acceleration is a function of position
 - Acceleration is *not* constant and therefore the uniformly accelerated motion equation cannot be applied

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Elastic Potential Energy

- A compressed spring has potential energy
 - The compressed spring, when allowed to expand, can apply a force to an object
 - The potential energy of the spring can be transformed into kinetic energy of the object

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Elastic Potential Energy, cont

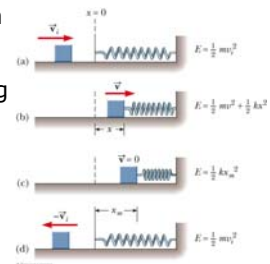
- The energy stored in a stretched or compressed spring or other elastic material is called *elastic potential energy*
 - $PE_s = \frac{1}{2}kx^2$
- The energy is stored only when the spring is stretched or compressed
- Elastic potential energy can be added to the statements of Conservation of Energy and Work-Energy

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Energy in a Spring Mass System

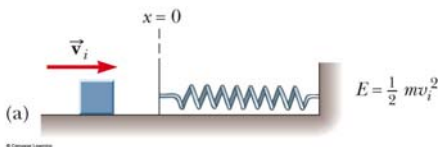
Please replace with fig. 13.4

- A block sliding on a frictionless system collides with a light spring
- The block attaches to the spring
- The system oscillates in Simple Harmonic Motion



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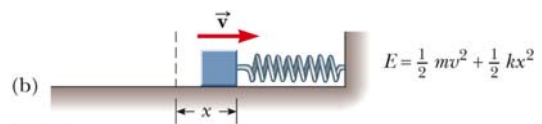
Energy Transformations



- The block is moving on a frictionless surface
- The total mechanical energy of the system is the kinetic energy of the block

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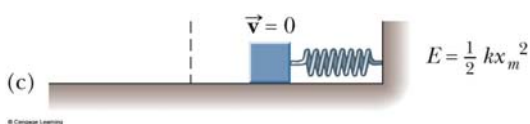
Energy Transformations, 2



- The spring is partially compressed
- The energy is shared between kinetic energy and elastic potential energy
- The total mechanical energy is the sum of the kinetic energy and the elastic potential energy

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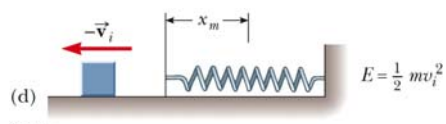
Energy Transformations, 3



- The spring is now fully compressed
- The block momentarily stops
- The total mechanical energy is stored as elastic potential energy of the spring

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Energy Transformations, 4



- When the block leaves the spring, the total mechanical energy is in the kinetic energy of the block
- The spring force is conservative and the total energy of the system remains constant

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Velocity as a Function of Position

- Conservation of Energy allows a calculation of the velocity of the object at any position in its motion

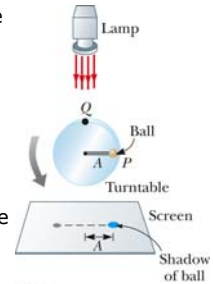
$$v = \pm \sqrt{\frac{k}{m}(A^2 - x^2)}$$

- Speed is a maximum at $x = 0$
- Speed is zero at $x = \pm A$
- The \pm indicates the object can be traveling in either direction

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Simple Harmonic Motion and Uniform Circular Motion

- A ball is attached to the rim of a turntable of radius A
- The focus is on the shadow that the ball casts on the screen
- When the turntable rotates with a constant angular speed, the shadow moves in simple harmonic motion



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Period and Frequency from Circular Motion

- Period $T = 2\pi\sqrt{\frac{m}{k}}$
 - This gives the time required for an object of mass m attached to a spring of constant k to complete one cycle of its motion
- Frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
 - Units are cycles/second or Hertz, Hz

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Angular Frequency

- The angular frequency is related to the frequency

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

- The *frequency* gives the number of cycles per second
- The *angular frequency* gives the number of radians per second

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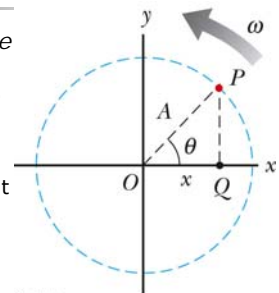
Effective Spring Mass

- A graph of T^2 versus m does not pass through the origin
- The spring has mass and oscillates
- For a cylindrical spring, the *effective* additional mass of a light spring is $1/3$ the mass of the spring

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Motion as a Function of Time

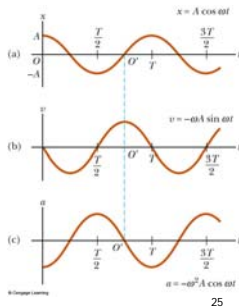
- Use of a *reference circle* allows a description of the motion
- $x = A \cos(2\pi ft)$
 - x is the position at time t
 - x varies between $+A$ and $-A$



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Graphical Representation of Motion

- When x is a maximum or minimum, velocity is zero
- When x is zero, the velocity is a maximum
- When x is a maximum in the positive direction, a is a maximum in the negative direction



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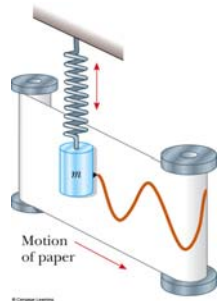
Motion Equations

- Remember, the uniformly accelerated motion equations cannot be used
- $x = A \cos (2\pi ft) = A \cos \omega t$
- $v = -2\pi f A \sin (2\pi ft) = -A \omega \sin \omega t$
- $a = -4\pi^2 f^2 A \cos (2\pi ft) = -A \omega^2 \cos \omega t$

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Verification of Sinusoidal Nature

- This experiment shows the sinusoidal nature of simple harmonic motion
- The spring mass system oscillates in simple harmonic motion
- The attached pen traces out the sinusoidal motion

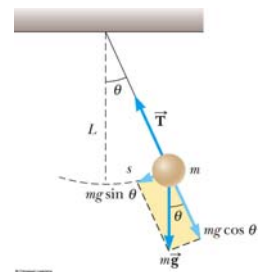


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Simple Pendulum

- The simple pendulum is another example of simple harmonic motion
- The force is the component of the weight tangent to the path of motion
 - $F_t = -mg \sin \theta$



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Simple Pendulum, cont

- In general, the motion of a pendulum is not simple harmonic
- However, for small angles, it becomes simple harmonic
 - In general, angles $< 15^\circ$ are small enough
 - $\sin \theta = \theta$
 - $F_t = -mg \theta$
 - This force obeys Hooke's Law

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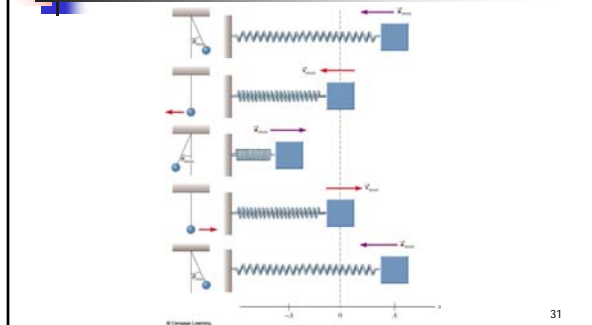
Period of Simple Pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

- This shows that the period is independent of the amplitude
- The period depends on the length of the pendulum and the acceleration of gravity at the location of the pendulum

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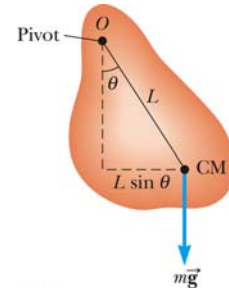
Simple Pendulum Compared to a Spring-Mass System



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Physical Pendulum

- A physical pendulum can be made from an object of any shape
- The center of mass oscillates along a circular arc



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Period of a Physical Pendulum

- The period of a physical pendulum is given by

$$T = 2\pi \sqrt{\frac{I}{mgL}}$$

- I is the object's moment of inertia
- m is the object's mass
- For a simple pendulum, $I = mL^2$ and the equation becomes that of the simple pendulum as seen before

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Damped Oscillations

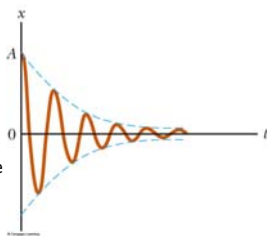
- Only ideal systems oscillate indefinitely
- In real systems, friction retards the motion
- Friction reduces the total energy of the system and the oscillation is said to be *damped*

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Damped Oscillations, cont.

- Damped motion varies depending on the fluid used

- With a low viscosity fluid, the vibrating motion is preserved, but the amplitude of vibration decreases in time and the motion ultimately ceases
 - This is known as *underdamped* oscillation



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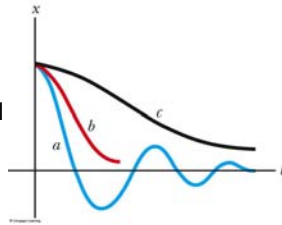
More Types of Damping

- With a higher viscosity, the object returns rapidly to equilibrium after it is released and does not oscillate
 - The system is said to be *critically damped*
- With an even higher viscosity, the piston returns to equilibrium without passing through the equilibrium position, but the time required is longer
 - This is said to be *overdamped*

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Graphs of Damped Oscillators

- Plot a shows an underdamped oscillator
- Plot b shows a critically damped oscillator
- Plot c shows an overdamped oscillator



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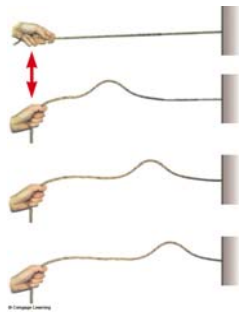
Wave Motion

- A wave is the motion of a disturbance
- Mechanical waves require
 - Some source of disturbance
 - A medium that can be disturbed
 - Some physical connection between or mechanism through which adjacent portions of the medium influence each other
- All waves carry energy and momentum

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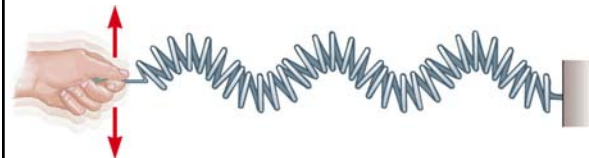
Types of Waves – Traveling Waves

- Flip one end of a long rope that is under tension and fixed at one end
- The pulse travels to the right with a definite speed
- A disturbance of this type is called a *traveling wave*



Types of Waves – Transverse

- In a transverse wave, each element that is disturbed moves in a direction perpendicular to the wave motion



(a) Transverse wave

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Types of Waves – Longitudinal

- In a longitudinal wave, the elements of the medium undergo displacements parallel to the motion of the wave
- A longitudinal wave is also called a compression wave



(b) Longitudinal wave

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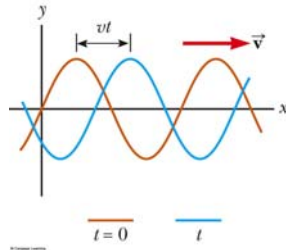
Other Types of Waves

- Waves may be a combination of transverse and longitudinal
- A **soliton** consists of a solitary wave front that propagates in isolation
 - First studied by John Scott Russell in 1849
 - Now used widely to model physical phenomena

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Waveform – A Picture of a Wave

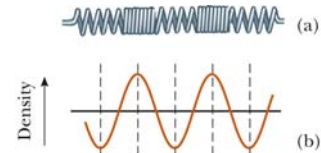
- The brown curve is a “snapshot” of the wave at some instant in time
- The blue curve is later in time
- The high points are *crests* of the wave
- The low points are *troughs* of the wave



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Longitudinal Wave Represented as a Sine Curve

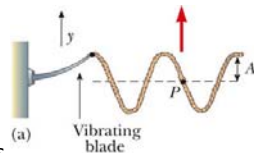
- A longitudinal wave can also be represented as a sine curve
- Compressions correspond to crests and stretches correspond to troughs
- Also called density waves or pressure waves



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Description of a Wave

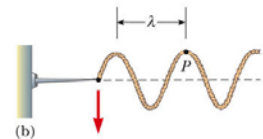
- A steady stream of pulses on a very long string produces a continuous wave
- The blade oscillates in simple harmonic motion
- Each small segment of the string, such as P, oscillates with simple harmonic motion



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Amplitude and Wavelength

- Amplitude is the maximum displacement of string above the equilibrium position
- Wavelength, λ , is the distance between two successive points that behave identically



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Speed of a Wave

- $v = f\lambda$
 - Is derived from the basic speed equation of distance/time
- This is a general equation that can be applied to many types of waves

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Speed of a Wave on a String

- The speed on a wave stretched under some tension, F

$$v = \sqrt{\frac{F}{\mu}} \text{ where } \mu = \frac{m}{L}$$

- μ is called the linear density
- The speed depends only upon the properties of the medium through which the disturbance travels

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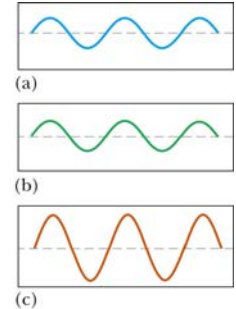
Interference of Waves

- Two traveling waves can meet and pass through each other without being destroyed or even altered
- Waves obey the *Superposition Principle*
 - When two or more traveling waves encounter each other while moving through a medium, the resulting wave is found by adding together the displacements of the individual waves point by point
 - Actually only true for waves with small amplitudes

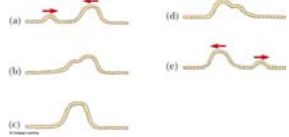
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Constructive Interference

- Two waves, a and b, have the same frequency and amplitude
 - Are *in phase*
- The combined wave, c, has the same frequency and a greater amplitude



Constructive Interference in a String

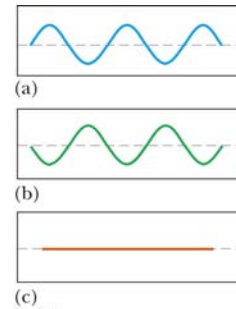


- Two pulses are traveling in opposite directions
- The net displacement when they overlap is the sum of the displacements of the pulses
- Note that the pulses are unchanged after the interference

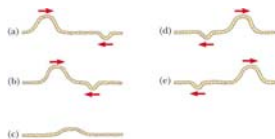
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Destructive Interference

- Two waves, a and b, have the same amplitude and frequency
 - They are 180° out of phase
- When they combine, the waveforms cancel



Destructive Interference in a String

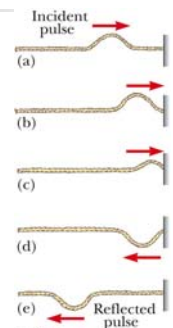


- Two pulses are traveling in opposite directions
- The net displacement when they overlap is decreased since the displacements of the pulses subtract
- Note that the pulses are unchanged after the interference

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Reflection of Waves – Fixed End

- Whenever a traveling wave reaches a boundary, some or all of the wave is reflected
- When it is reflected from a fixed end, the wave is inverted
- The shape remains the same





Reflected Wave – Free End

- When a traveling wave reaches a boundary, all or part of it is reflected
- When reflected from a free end, the pulse is not inverted

