Chapter 7 - Solutions

Rotational Motion and the Law of Gravity

PROBLEM SOLUTIONS

7.3 (a) $\theta = \frac{s}{r} = \frac{60\,000 \text{ mi}}{1.0 \text{ ft}}\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right) = \boxed{3.2\times10^{8} \text{ rad}}$

(b) $\theta = 3.2\times10^{8} \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{5.0\times10^{7} \text{ rev}}$

7.6 $\omega_i = 3\,600 \ \frac{\text{rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right) = 377 \text{ rad/s}$

$\Delta\theta = 50.0 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 314 \text{ rad}$

Thus,

$$\alpha = \frac{w^2 - \omega_i^2}{2\Delta\theta} = \frac{0-(377 \text{ rad/s})^2}{2(314 \text{ rad})} = \boxed{-226 \text{ rad/s}^2}$$

7.12 (a) The angular speed is $\omega = \omega_0 + \alpha t = 0 + (2.50 \text{ rad/s}^2)(2.30 \text{ s}) = \boxed{5.75 \text{ rad/s}}$.

(b) Since the disk has a diameter of 45.0 cm, its radius is r = (00.450 m)/2 = 0.225 m. Thus,

$$v_t = r\omega = (0.225 \text{ m})(5.75 \text{ rad/s}) = \boxed{1.29 \text{ m/s}}$$

and

$$a_t = r\alpha = (0.225 \text{ m})(2.50 \text{ rad/s}^2) = \boxed{0.563 \text{ m/s}^2}$$

(c) The angular displacement of the disk is

$$\Delta\theta = \theta_f - \theta_0 = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{(5.75\ \text{rad/s})^2 - 0}{2(2.50\ \text{rad/s}^2)} = (6.61\ \text{rad})\left(\frac{360°}{2\pi\ \text{rad}}\right) = 379°$$

and the final angular position of the radius line through point P is

$$\theta_f = \theta_0 + \Delta\theta = 57.3° + 379° = 436°$$

or it is at $\boxed{76° \text{ counterclockwise from the } +x\text{-axis}}$ after turning 19° beyond one full revolution.

7.24 Since $F_c = m\frac{v_t^2}{r} = m\,r\,\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{mr}} = \sqrt{\frac{4.0 \times 10^{-11}\ \text{N}}{(3.0 \times 10^{-16}\ \text{kg})(0.150\ \text{m})}}$$

$$= (9.4 \times 10^2\ \text{rad/s})\left(\frac{1\ \text{rev}}{2\,\pi\ \text{rad}}\right) = \boxed{1.5 \times 10^2\ \text{rev/s}}$$

7.36 (a) The density of the white dwarf would be

$$\rho = \frac{M}{V} = \frac{M_{\text{sun}}}{V_{\text{Earth}}} = \frac{M_{\text{sun}}}{4\pi R_E^3/3} = \frac{3M_{\text{sun}}}{4\pi R_E^3}$$

and using data from Table 7.3,

$$\rho = \frac{3(1.991 \times 10^{30}\ \text{kg})}{4\pi(6.38 \times 10^6\ \text{m})^3} = \boxed{1.83 \times 10^9\ \text{kg/m}^3}$$

(b) $F_g = mg = GMm/r^2$, so the acceleration of gravity on the surface of the white dwarf would be

$$g = \frac{GM_{\text{sun}}}{R_E^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.991 \times 10^{30}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2} = \boxed{3.26 \times 10^6\ \text{m/s}^2}$$

(c) The general expression for the gravitational potential energy of an object of mass m at distance r from the center of a spherical mass M is $PE = -GMm/r$. Thus, the potential energy of a 1.00-kg mass on the surface of the white dwarf would be

$$PE = -\frac{GM_{\text{sun}}(1.00\ \text{kg})}{R_E}$$
$$= -\frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.991\times 10^{30}\ \text{kg})(1.00\ \text{kg})}{6.38\times 10^{6}\ \text{m}} = \boxed{-2.08\times 10^{13}\ \text{J}}$$

7.42 For an object in orbit about Earth, Kepler's third law gives the relation between the orbital period *T* and the average radius of the orbit ("semi-major axis") as

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3$$

Thus, if the average radius is

$$r = \frac{r_{\text{min}} + r_{\text{max}}}{2} = \frac{6\ 670\ \text{km} + 385\ 000\ \text{km}}{2} = 1.96\times 10^{5}\ \text{km} = 1.96\times 10^{8}\ \text{m}$$

the period (time for a round trip from Earth to the Moon) would be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{(1.96\times 10^{8}\ \text{m})^3}{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\ \text{kg})}} = 8.63\times 10^{5}\ \text{s}$$

The time for a one way trip from Earth to the Moon is then

$$\Delta t = \frac{1}{2}T = \frac{8.63\times 10^{5}\ \text{s}}{2}\left(\frac{1\ \text{day}}{8.64\times 10^{4}\ \text{s}}\right) = \boxed{4.99\ \text{d}}$$