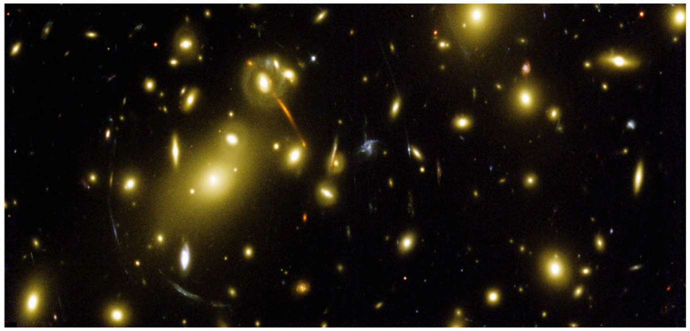
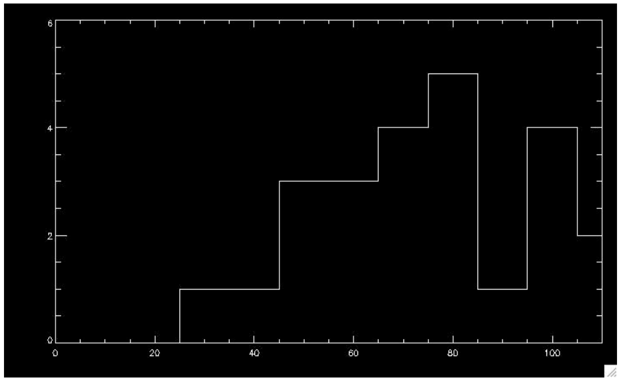
Lecture 18 Spacetime and Gravity
A2020 Prof. Tom Megeath



Midterm 2 Grade Distribution



Review: Inertial Reference Frames

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Speed limit sign posted on spacestation. How fast is that man moving?

The Solar System is orbiting our Galaxy at 220 km/s. Do you feel this?

Imagine two spaceships passing. The astronaut on each spaceship thinks that he is stationary and that the other spaceship is moving.

Which one is right? Both. Each one is an inertial reference frame.

Any non-rotating reference frame is an inertial reference frame (space shuttle, space station). Each reference frame is equally valid.

In contrast, you can tell if a reference frame is rotating.

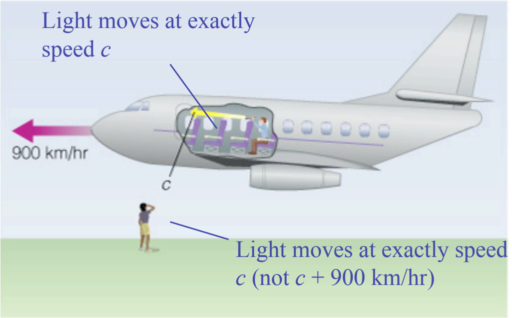
Review: Absolutes of Relativity

1. The laws of nature are the same for everyone
2. The speed of light is the same for everyone

All of relativity follows from these two ideas!

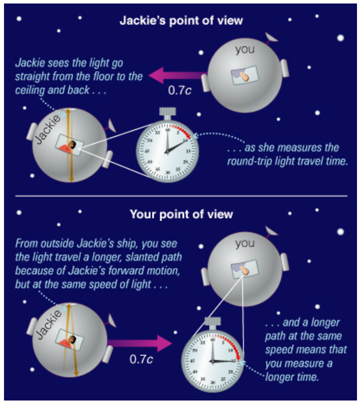
<u>However, we have to abandon the idea that time is absolute!</u>

Review: Absoluteness of Light Speed



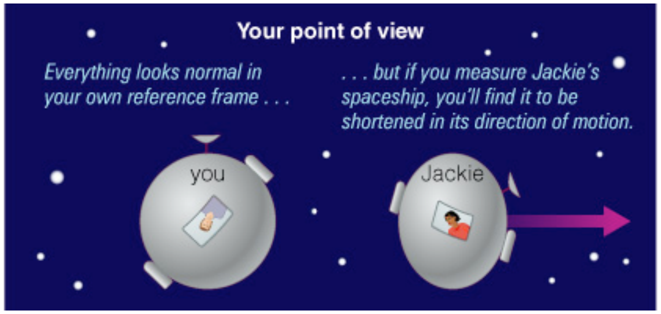
- Einstein claimed that light should move at exactly *c* in all reference frames (now experimentally verified)

Review: Time Dilation



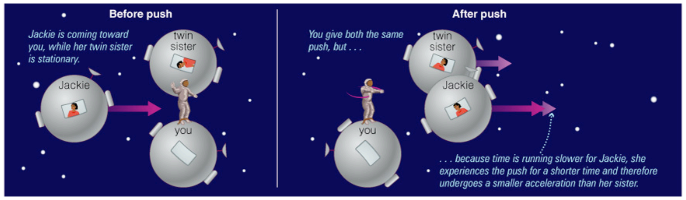
- We can perform a thought experiment with a light beam replacing the ball
- The light beam, moving at *c*, travels a longer path in a moving object
- Time must be passing more slowly there

Review: Length Contraction



- Similar thought experiments tell us that an object's length becomes shorter in its direction of motion

Review: Mass Increase



- A force applied to a rapidly moving object produces less acceleration than if the object were motionless
- This effect can be attributed to a mass increase in the moving object (Remember: Force = Mass x Acceleration)

Review: Formulas of Special Relativity

Time Dilation: $t' = t\sqrt{1-\left(\frac{v^2}{c^2}\right)}$

Length Contraction: $l' = l\sqrt{1-\left(\frac{v^2}{c^2}\right)}$

Mass Increase: $m' = \frac{m}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$

Review: Deriving $E = mc^2$

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \approx m_0\left(1+\frac{1}{2}\frac{v^2}{c^2}\right)$$ for small v

$$\text{Total energy} = mc^2 \approx m_0c^2 + \frac{1}{2}m_0v^2$$

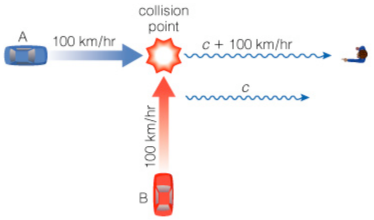
Mass-Energy of object at rest

Kinetic Energy

Tests of Special Relativity

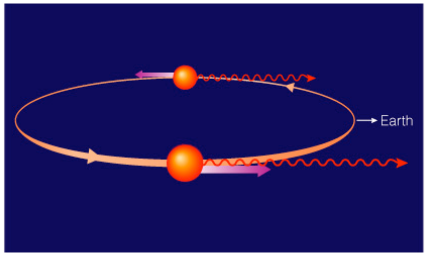
- First evidence for absoluteness of speed of light came from the *Michaelson-Morley Experiment* performed in 1887
- Time dilation happens routinely to subatomic particles the approach the speed of light in accelerators
- Time dilation has also been verified through precision measurements in airplanes moving at much slower speeds
- Prediction that $E=mc^2$ is verified daily in nuclear reactors and in the core of the Sun

A Paradox of Non-Relativistic Thinking



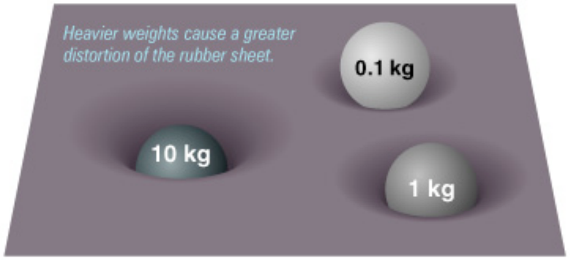
- If speed of light were not absolute, you would see the car coming toward you reach the collision point before the car it struck
- No paradox if light speed is same for everyone

Test Relativity for Yourself



- If speed of light were not absolute, binary stars would not look like two distinct points of light
- You can verify relativity by simply looking through a telescope at a binary star system

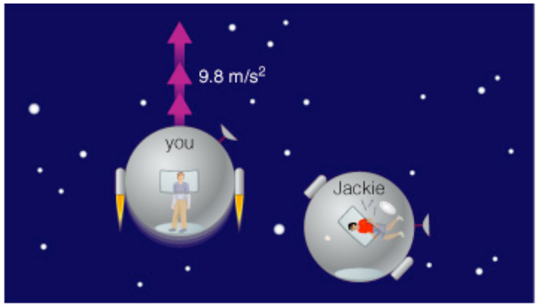
What are the major ideas of General relativity?



Spacetime

- Special relativity showed that space and time are not absolute
- Instead they are inextricably linked in a four-dimensional combination called **spacetime**
- In general relativity we find that acceleration and gravity are equivalent
- That space and time are curved.

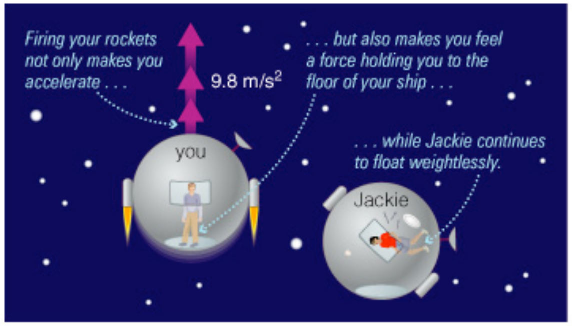
Is all motion relative?



Relativity and Acceleration

- Our thought experiments about special relativity involved spaceships moving at constant velocity
- Is all motion still relative when acceleration and gravity enter the picture?

Acceleration and Relative Motion



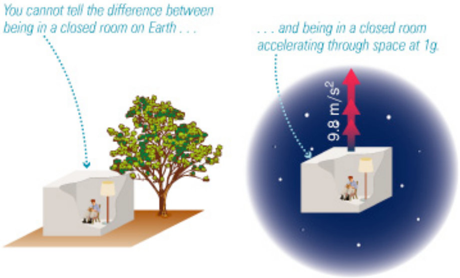
Firing your rockets not only makes you accelerate . . .

. . . but also makes you feel a force holding you to the floor of your ship . . .

. . . while Jackie continues to float weightlessly.

- How can your motion be relative if you're feeling a force causing acceleration?

The Equivalence Principle

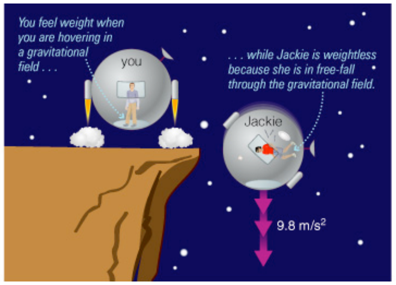


You cannot tell the difference between being in a closed room on Earth . . .

. . . and being in a closed room accelerating through space at 1g.

- Einstein preserved the idea that all motion is relative by pointing out that the effects of acceleration are exactly equivalent to those of gravity

Gravity and Relative Motion



You feel weight when you are hovering in a gravitational field . . .

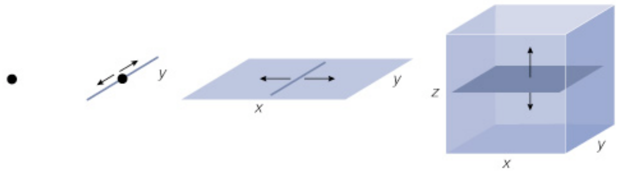
. . . while Jackie is weightless because she is in free-fall through the gravitational field.

- Someone who feels a force may be hovering in a gravitational field
- Someone who feels weightless may be in free-fall

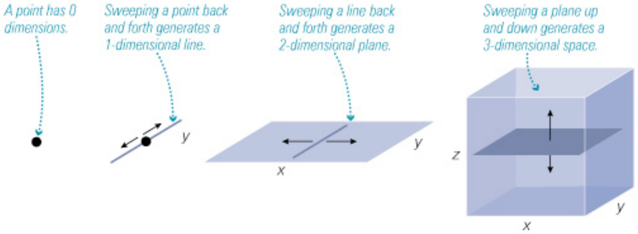
Einstein's *Equivalence Principle*

The effects of acceleration are equivalent to the effects of gravity

What is spacetime?



Dimensions of Space

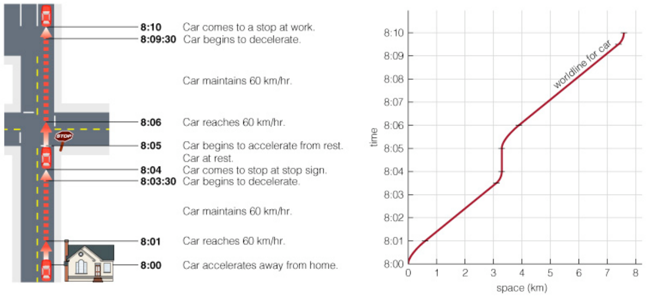


- An object's number of dimensions is the number of independent directions in which movement is possible within the object

Dimensions of Spacetime

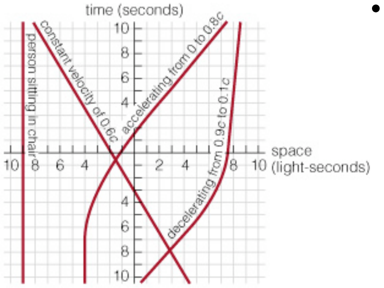
- We can move through three dimensions in space (x,y,z)
- Our motion through time is in one direction (t)
- Spacetime, the combination of space and time, has four dimensions (x,y,x,t)

Spacetime Diagram of a Car (x,t)



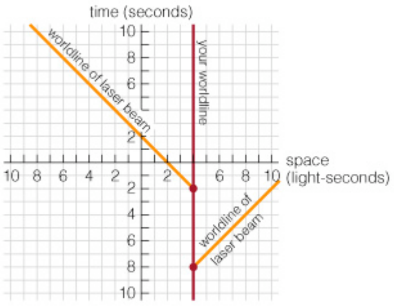
- A spacetime diagram plots an object's position in space at different moments in time

Worldlines (x,t)



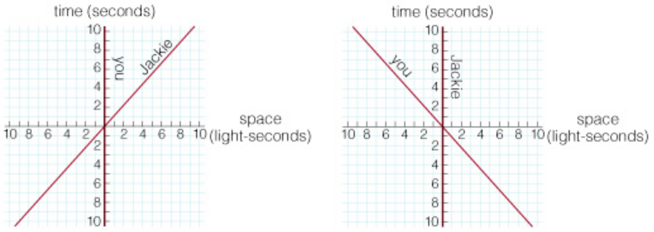
- A worldline shows an object's path through spacetime in a spacetime diagram
  - Vertical worldline: no motion
  - Diagonal worldline: constant-velocity motion
  - Curved worldline: accelerating motion

Worldlines for Light (x,t)



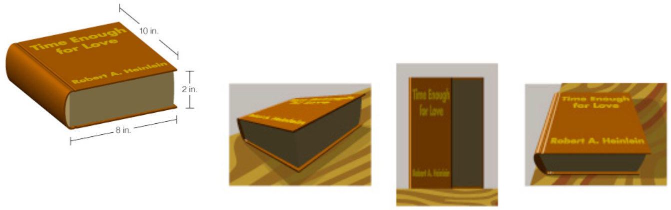
- Worldlines for light go at 45° angles in diagrams with light-seconds on one axis and seconds on the other

Worldlines and Relativity



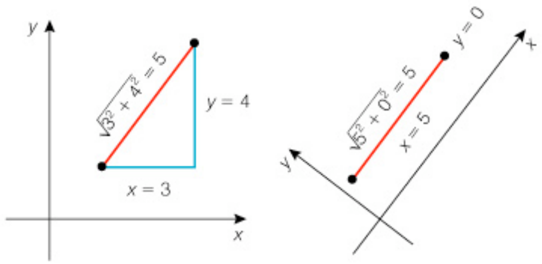
- Worldlines look different in different reference frames

Perspectives in Space



- A book has a definite three-dimensional shape
- But the book looks different in two-dimensional pictures of the book taken from different perspectives
- Similarly, space and time look different from different reference frames in spacetime

Different Coordinate Systems



Imagine two surveyors mapping a property. They both a different coordinate grid. They want to measure the distance between two trees.

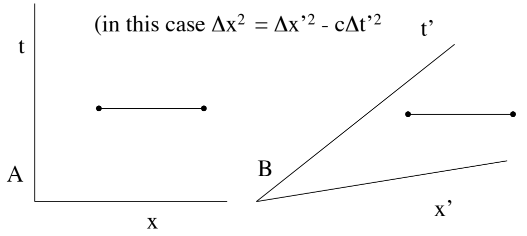
What is constant: the distance $x^2+y^2 = x'^2+y'^2$

What are not constant - x, y

Different Spacetime Coordinate Systems for Different Inertial Reference Frames

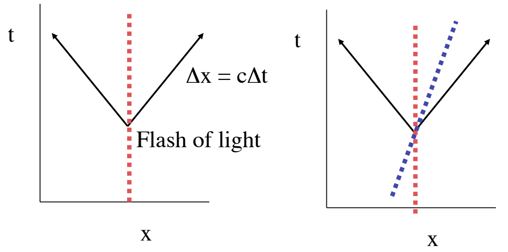
$\Delta x^2 - c\Delta t^2 = \Delta x'^2 - c\Delta t'^2$

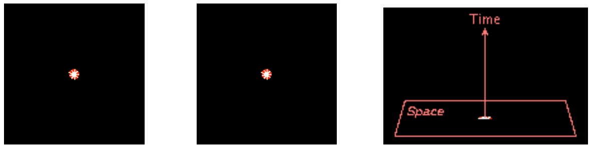
(in this case $\Delta x^2 = \Delta x'^2 - c\Delta t'^2$



- Two observers move past each other.
- Both have their own spacetime coordinate system
- They measure the same event.
- Observer A thinks the events are simultaneous
- Observe B does not.
- They can agree on one thing "interval" between two different events in spacetime: $x^2 + y^2 + z^2 - (ct)^2$

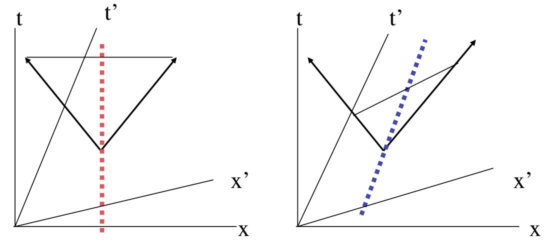
Light Cones



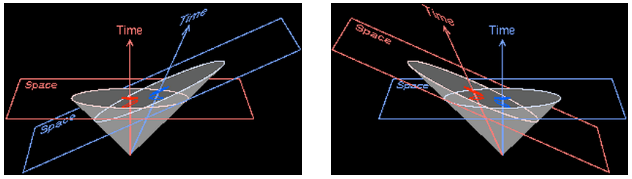


Andrew Hamilton http://casa.colorado.edu/~ajsh/sr/sr.shtml (excellent web site!!)

Solution



Two events which are simultaneous in one reference frame are not simultaneous in the other frame.

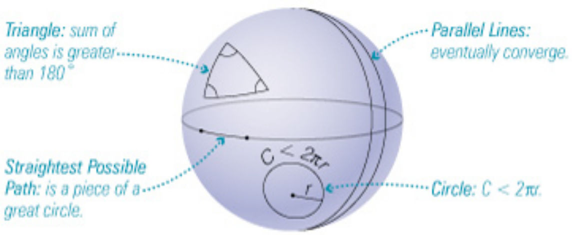


Spacetime

- For a surveyor on Earth. You need to consider two directions: North-South, East-West. *These two directions cannot be considered separate entities*, but parts of a single coordinate system.
- in relative motion do not share the same definitions of *x*, *y*, *z*, and *t*, taken individually

Space is different for different observers.
Time is different for different observers.
To account for these differences, we must consider spacetime a single entity

What is curved spacetime?



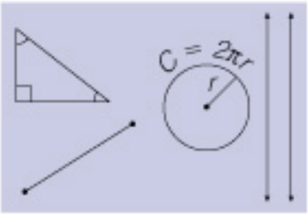
Curved 2-Dimentional Space

- Travelers going in opposite directions in straight lines will eventually meet
- Because they meet, the travelers know Earth's surface cannot be flat—it must be curved

Curved 4-Dimensional Spacetime

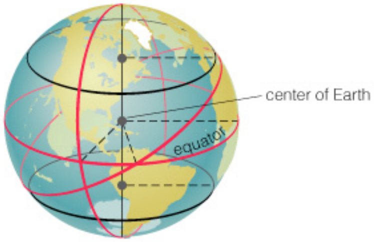
- Gravity can cause two space probes moving around Earth to meet
- General relativity says this happens because spacetime is curved

Rules of Geometry in Flat Space



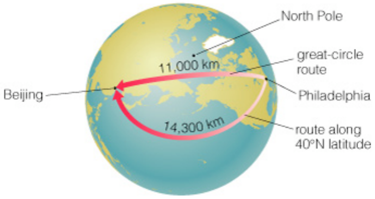
- Straight line is shortest distance between two points
- Parallel lines stay same distance apart
- Angles of a triangle sum to 180°
- Circumference of circle is $2\pi r$

Geometry on a Curved Surface



- The straightest lines on a sphere are *great circles* sharing the same center as the sphere
- Great circles intersect, unlike parallel lines in flat space

Geometry on a Curved Surface

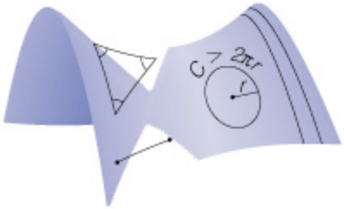


- Straight lines are shortest paths between two points in flat space
- Great circles are the shortest paths between two points on a sphere

Rules of Spherical Geometry

- Great circle is shortest distance between two points
- Parallel lines eventually converge
- Angles of a triangle sum to > 180°
- Circumference of circle is $< 2\pi r$

Rules of Saddle-Shaped Geometry



- Piece of hyperbola is shortest distance between two points
- Parallel lines diverge
- Angles of a triangle sum to < 180°
- Circumference of circle is $> 2\pi r$

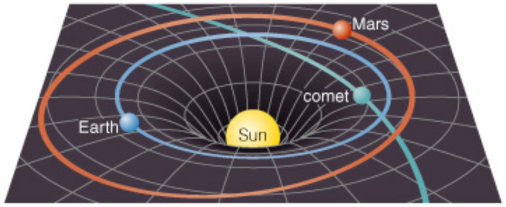
“Straight” lines in Spacetime

- According to Equivalence Principle:
  - If you are floating freely, then your worldline is following the *straightest possible path* through spacetime
  - If you feel weight, then you are not on the straightest possible path – you are accelerating.

What have we learned?

- What is spacetime?
  - Spacetime is the four-dimensional combination of space and time that forms the “fabric” of our universe
- What is curved spacetime?
  - Spacetime can be curved just as a piece of paper can be curved
  - Three example geometries for spacetime are flat, spherical, and saddle-shaped
  - The rules of geometry differ among these cases

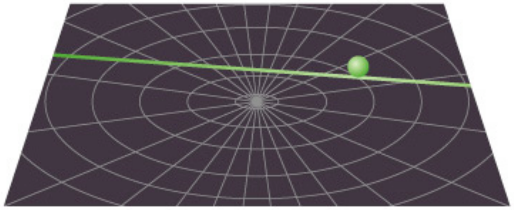
What is gravity?



Gravity, Newton, and Einstein

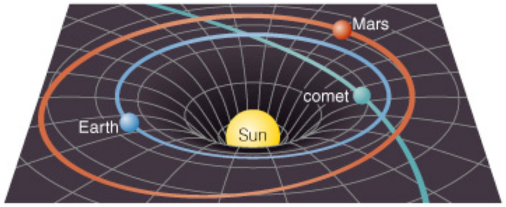
- Newton viewed gravity as a mysterious “action at a distance”
- Einstein removed the mystery by showing that what we perceive as gravity arises from curvature of spacetime

Rubber Sheet Analogy



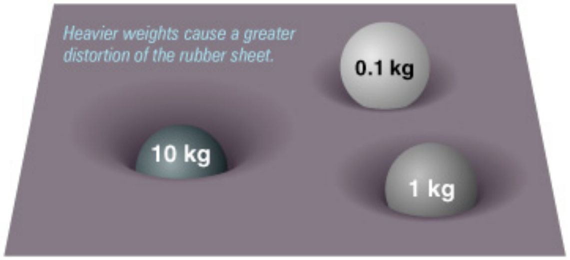
- On a flat rubber sheet
  - Free-falling objects move in straight lines
  - Circles all have circumference $2\pi r$

Rubber Sheet Analogy



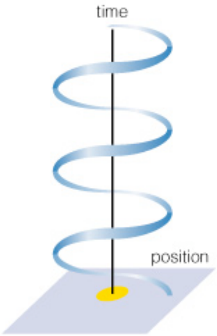
- Mass of Sun curves spacetime
  - Free-falling objects near Sun follow curved paths
  - Circles near Sun have circumference $< 2\pi r$

Rubber Sheet Analogy



- Matter distorts spacetime in a manner analogous to how heavy weights distort a rubber sheet

Limitations of the Analogy

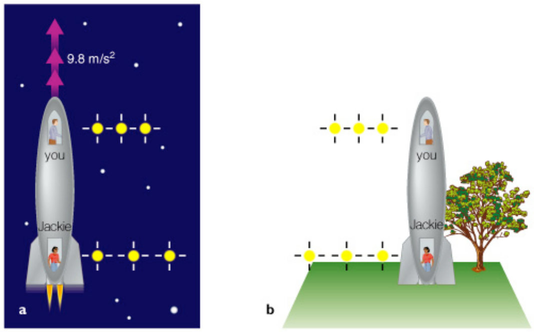


- Masses do not rest "upon" the spacetime like they rest on a rubber sheet
- Rubber sheet shows only two dimensions of space
- Rubber sheet shows only two dimensions of space
- Path of an orbiting object actually spirals through spacetime as it moves forward in time

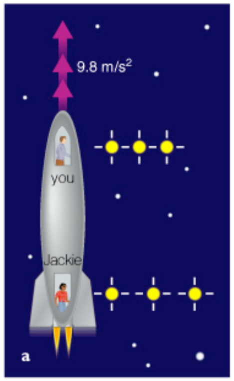
Basics of General Relativity

- Matter & Energy tell spacetime how to curve.
- Curvature of spacetime tells matter and energy how to move.
- Particles, spaceships, light, etc, follow the locally straight path through curved spacetime
- If they don't, they are accelerating (i.e. the rocket engine is turned on).

How does gravity affect time?

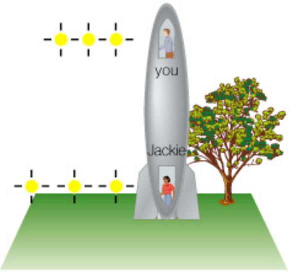


Time in an Accelerating Spaceship



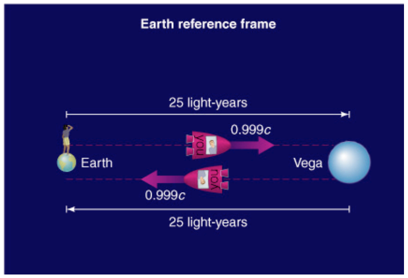
- Light pulse travel more quickly from front to back of an accelerating spaceship than in other direction
- Everyone on ship agrees that time runs faster in front than in back

Time in an Gravitational Field



- Effects of gravity are exactly equivalent to those of acceleration
- Time must run more quickly at higher altitudes in a gravitational field than at lower altitudes

A Journey to Vega



- The distance to Vega is about 25 light-years
- But if you could travel to Vega at 0.999*c*, the round trip would seem to take only two years!
- At that speed, the distance to Vega contracts to only 1 light-year in your reference frame
- Your twin on Earth would have aged 50 years while you aged only 2
- There's a seeming contradiction to this conclusion:What does your twin see in his reference frame as he watches the Earth recede?

An alternative solution of the Twin Paradox

- If one twin takes a high-speed round trip to a distant star, that twin will have aged less than the other that remains on Earth
- But doesn't time on Earth appear to run slower from the perspective of the twin on the high-speed trip?
- Solution: The twin on the trip is accelerating

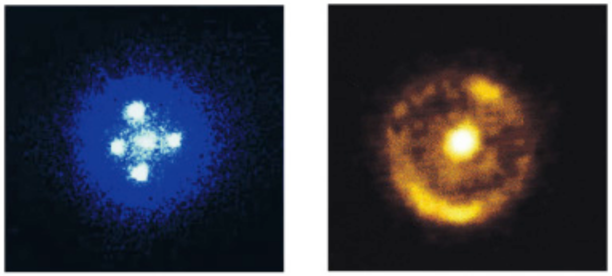
What have we learned?

- What is gravity?
  - Gravity arises from curvature of spacetime
  - Matter and energy tell spacetime how to curve
  - Spacetime tells matter and energy how to move.
  - Objects take the locally straightest path in curved space
  - If they don't, they are accelerating.
- How does gravity affect time?
  - Time runs more slowly at lower altitudes in a gravitational field

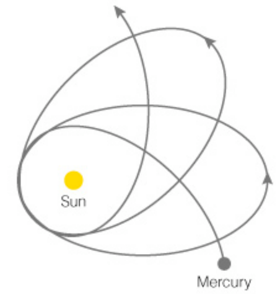
A Relativity Puzzle

Imagine you have two spaceships, one in deep space (flat spacetime) and one orbiting the earth (curved spacetime). The spaceships have no windows. Can the astronauts tell whether they are floating in deep space or orbiting the Earth?

How do we test the predictions of general relativity?



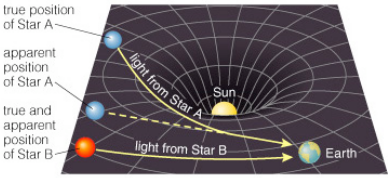
Precession of Mercury



Note: The amount of precession with each orbit is highly exaggerated in this picture.

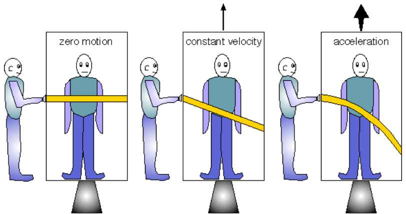
- The major axis of Mercury's elliptical orbit precesses with time at a rate that disagrees with Newton's laws
- General relativity precisely accounts for Mercury's precession

Gravitational Lensing



- Curved spacetime alters the paths of light rays, shifting the apparent positions of objects in an effect called *gravitational lensing*

Bending of Light and the Equivalence Principle



The path of a light beam in three different types of reference frames moving with respect to the person *outside* the elevator. The light path shown is what the person ***inside*** the elevator sees. Under large acceleration, the beam of light will curve downward. It should also do that in a region of strong gravity.

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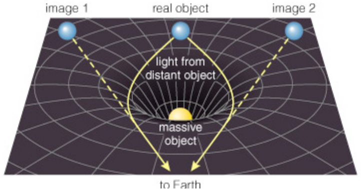
Gravitational Lensing



- Gravitational lensing can distort the images of objects
- Lensing can even make one object appear to be at two or more points in the sky

The First Measurement of the Deflection of Starlight

Einstein predicted that the Sun would bend the starlight from distant stars as they passed behind the Sun.

Measuring this required that a star be observed near the Sun

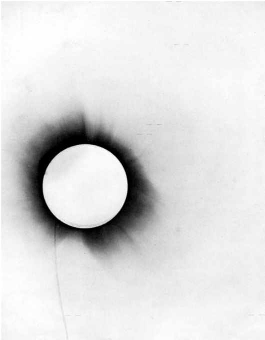
A solar eclipse was expected in 1917

Sir Arthur Eddington mounted an expedition to take observations at two locations: the island of Principe, Gulf of Guinea, West Africa and Sobral in Brazil.

Telescopes had to be brought to the sites and assembled.

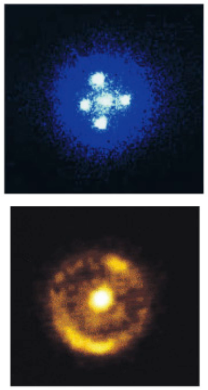
Despite marginal weather at one of the sites, and poor observations at the other, the deflection of starlight was observed.

The announcement of results resulted in Einstein's fame



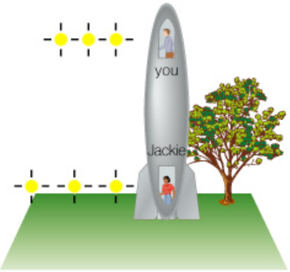
Photographic plate used to measure deflection.

Gravitational Lensing



- Gravity of foreground galaxy (center) bends light from an object almost directly behind it
- Four images of that object appear in the sky (Einstein's Cross)
- Gravity of foreground galaxy (center) bends light from an object directly behind it
- A ring of light from the background object appears in the sky (Einstein Ring)

Gravitational Time Dilation

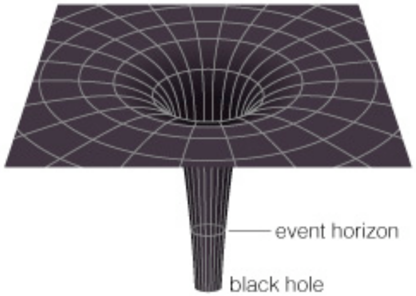


- Passage of time at different altitudes has been precisely measured
- Time indeed passes more slowly at lower altitudes in precise agreement with general relativity
- Needed to be taken into account for GPS satellites

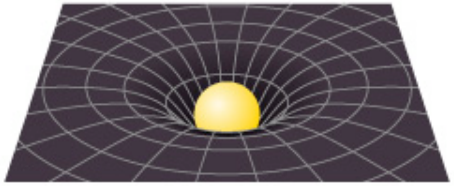
Testing Relativity

- How do we verify special relativity?
  - Absolute speed of light of came from the *Michaelson-Morley Experiment*
  - Time dilation measured for subatomic particles
  - Time dilation measured in airplanes
  - $E=mc^2$ verified in nuclear reactors and in Sun the core of the Sun
- How do we test the predictions of the general theory of relativity?
  - Precession of Mercury
  - Gravitational Lensing
  - Gravitational Time Dilation

What is a black hole?

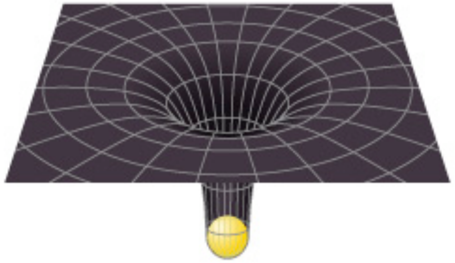


Curvature near Sun



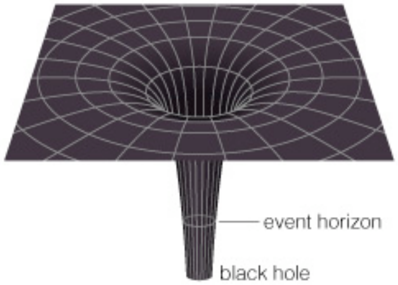
- Sun’s mass curves spacetime near its surface

Curvature near Sun



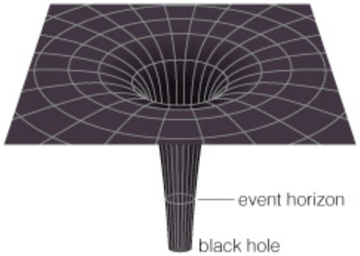
- If we could shrink the Sun without changing its mass, curvature of spacetime would become greater near its surface, as would strength of gravity

Curvature near Black Hole



- Continued shrinkage of Sun would eventually make curvature so great that it would be like a bottomless pit in spacetime: a *black hole*

The Event Horizon



- Spacetime is so curved near a black hole that nothing can escape
- The slope of the sheet become infinite
- The “point of no return” is called the *event horizon*
- Event horizon is a three-dimensional surface

What is a theory?

This is not the way science works.

Hypothesis → Theory→ Law (proven)

What is a scientific theory?

- The word theory has a different meaning in science than in everyday life.
- In science, a theory is NOT the same as a hypothesis, rather:
- A ***scientific theory*** is an explanation which connects a series of observations and facts through logical and mathematical analysis.
- A ***scientific theory*** must:
  - — Explain a wide variety of observations with a few simple principles, AND
  - — Must be supported by a large, compelling body of evidence.
  - — Must NOT have failed any crucial test of its validity (or have well understood limits to its validity)
  - — Must be able to make new predictions

Finally, a theory is not necessarily a finished work, and can be revised, extended, or superceded by a more complete theory.

Hallmarks of Scientific Theory: #1

Science theories are explanations for natural and repeatable phenomena

Modern scientific theories seeks explanations for observed phenomena that rely solely on natural causes which are consistent with logic, mathematics, and observations as best we understand them.

Repeatability essential: observed phenomena and natural causes must be repeatable for theories to be tested. An experiment or observation must be repeatable, or it is not to be believed and considered to be in error.

Hallmarks of a Scientific Theory: #2

Science progresses through the creation and testing of theories of nature that explain the observations as simply as possible .

Occam’s razor: from William of Occam, a 14th century English logician and franciscan friar

entia non sunt multiplicanda praeter necessitatem

(entities should not be multiplied beyond necessity)

Hallmarks of a Scientific : #3

Theories are not proven like a mathematical theorem. Their validity is *inferred* by the following:

1. Consistent with logic and mathematics
2. Consistent with observations and current knowledge of nature.
3. The ability to make testable predictions
4. **Most importantly**: that those predictions are confirmed in the laboratory or in observations of nature.

See discussion on deductive and inductive logic in Cosmic Perspective (page 81 chapter 3)

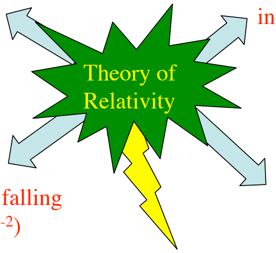
Accordingly, a theory is sort of a working model of nature, to be used as long as it proves useful, and improved or discarded when it no longer provides an adequate explanation.

> “A theory is a policy, not a creed”
> J. J. Thomson, discoverer of the electron

Scientific Laws

- Laws are mathematical relationship or principle that are generally - or for fundamental laws, always - true.
  - Example: Newton’s three laws
- Theories may contain laws
  - Example: Newton laws are the foundation of a theory of motion
- The word “law” is not commonly used anymore (more popular in the time of Newton)

Theories connect the dots by explaining the relationship between observations and then make new predictions



Observation: constant speed of light

Observation: Orbits of planets including Mercury

Theory of Relativity

Observations: acceleration of falling bodies (9.8 m s^{-2})

Observation: equivalence of acceleration and gravity

Predictions: Bending of Light (gravitational lensing)
Time Dilation (needed for GPS)
Black Holes

The ability of humans to develop theories that can make accurate predictions and extend our knowledge is both amazing and incredibly useful.

Even though you cannot disprove this, it is not a theory! Why?

Planets move in orbits because they are pushed by invisible, drunken, space elves with green hats.

Examples of Theories:

Heliocentric Theory of the Solar System
(predicted distances and orbital periods)

Newtonian Theory of Motion and Gravity
(predicted motions pretty much everything)

Theory of Relativity (Special and General)
(predicted orbit of Mercury, time dilation)

Electromagnetism
(predicted the existence of radio waves)

Quantum theory
(basis for lasers, semiconductors)

What is Scientific Theory?

A theory is an explanation of how nature works, often invoking mathematical laws (but this is not necessary).

- A good theory explains a wide variety of observations in terms of a few general principles
- A theory must make predictions to be valuable and testable
- A theory must be consistent with all observations and facts. Theories are constantly tested. If they fail, they are modified, incorporated into more general theories, or thrown out.
- However, certain theories are still useful even if there are known exceptions: example Newtonian physics.