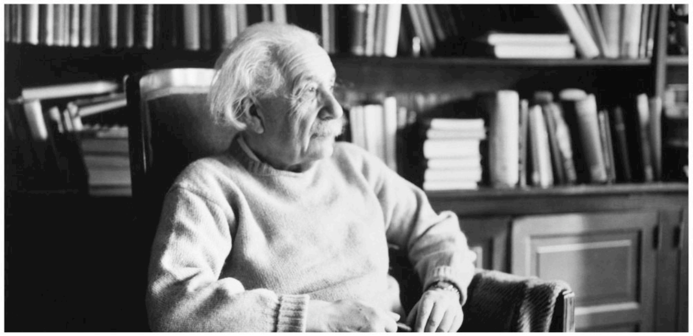
Lecture 17 Relativity
A2020 Prof. Tom Megeath



What are the major ideas of special relativity?

Einstein in 1921

(born 1879 - died 1955)

Einstein's Theories of Relativity

- Special Theory of Relativity (1905)
  - Usual notions of space and time must be revised for speeds approaching light speed (c)
  - $E = mc^2$
- General Theory of Relativity (1915)
  - Expands the ideas of special theory to include a surprising new view of gravity

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Key Ideas of Special Relativity

- No material object can travel faster than light
- If you observe something moving near light speed:
  - Its time slows down
  - Its length contracts in direction of motion
  - Its mass increases
- Whether or not two events are simultaneous depends on your perspective

Inertial Reference Frames

ISS006E45182

Speed limit sign posted on spacestation. How fast is that man moving?

The Solar System is orbiting our Galaxy at 220 km/s. Do you feel this?

Imagine two spaceships passing. The astronaut on each spaceship thinks that he is stationary and that the other spaceship is moving.

Which one is right? Both. Each one is an inertial reference frame.

Any non-rotating reference frame is an inertial reference frame (space shuttle, space station). Each reference frame is equally valid.

In contrast, you can tell if a reference frame is rotating.

Galilean Relativity

http://faraday.physics.utoronto.ca/PVB/Harrison/Flash/ClassMechanics/Relativity/Relativity.html

Absolute Time

In the Newtonian universe, time is absolute.

Thus, for any two people, reference frames, planets, etc, time marches along at the same rate.

If we the time between two events, that time is the same for every observer.

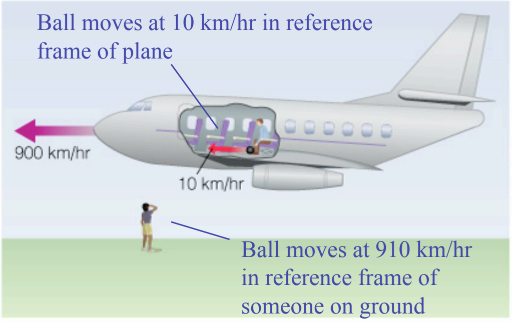
Absolutes of Relativity

1. The laws of nature are the same for everyone
2. The speed of light is the same for everyone

All of relativity follows from these two ideas!

However, we have to abandon the idea that time is absolute!

Reference Frames

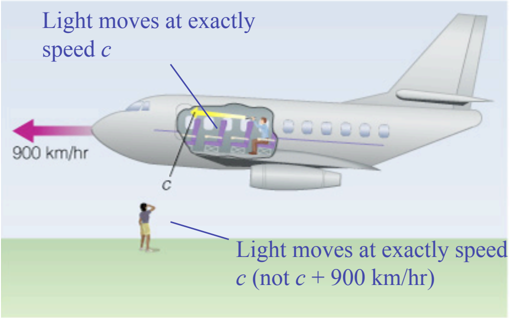


Ball moves at 10 km/hr in reference frame of plane

Ball moves at 910 km/hr in reference frame of someone on ground

- Motion can be defined with respect to a particular frame of reference

Absoluteness of Light Speed

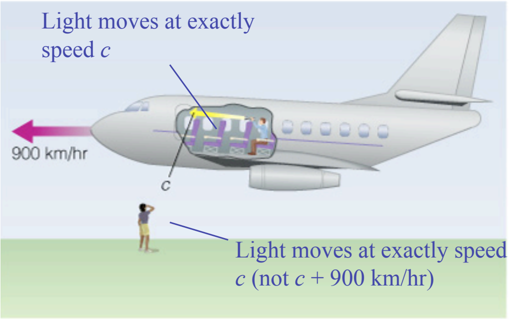


Light moves at exactly speed *c*

Light moves at exactly speed *c* (not *c* + 900 km/hr)

- Einstein claimed that light should move at exactly *c* in all reference frames (now experimentally verified)

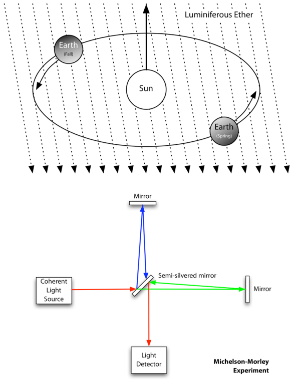
What's surprising about the absoluteness of the speed of light?



Light moves at exactly speed *c*

Light moves at exactly speed *c* (not *c* + 900 km/hr)

The Michaelson Morley Experiment



Originally thought that the speed of light was relative to some unknown medium, the aether.

This would be an absolute reference frame.

Michelson and Morely tried to measure the speed of the Earth through the aether. This was done at Case-Western Reserve University in 1817..

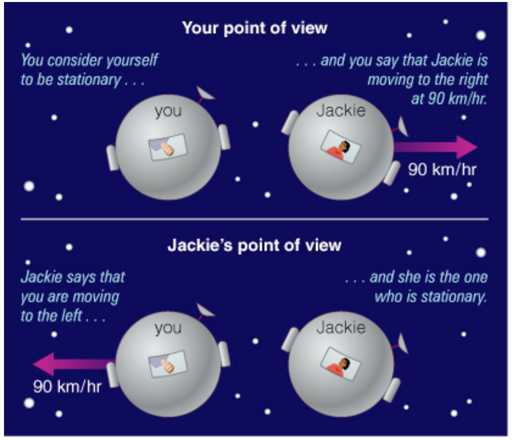
The could find no evidence for the aether.

http://www.upscale.utoronto.ca/PVB/Harrison/SpecRel/Flash/MichelsonMorley/MichelsonMorley.html

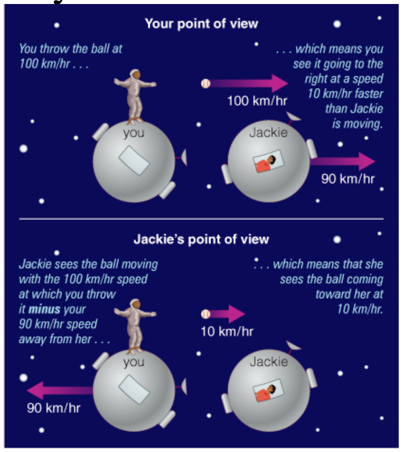
Thought (Gedanken) Experiments

- Einstein explored the consequences of the absoluteness of light speed using “thought experiments”
- The consequences will be easiest for us to visualize with thought experiments involving spaceships in freely floating reference frames (no gravity or acceleration)

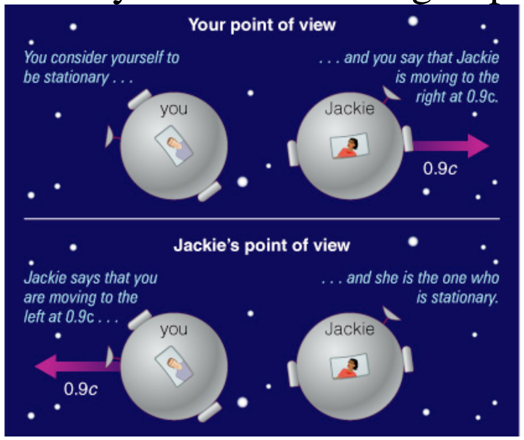
Relativity of Motion at Low Speeds



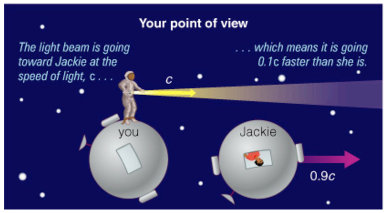
Relativity of Motion at Low Speeds



Relativity of Motion at High Speeds

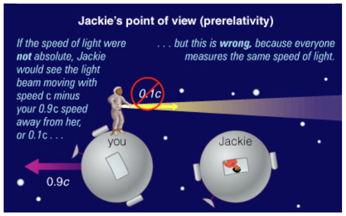


Light Speed is Absolute



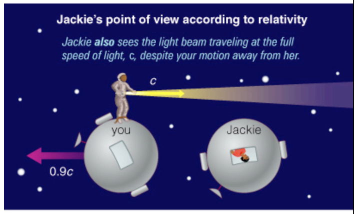
The light beam is going toward Jackie at the speed of light, c . . .

. . . which means it is going 0.1c faster than she is.



If the speed of light were ***not*** *absolute, Jackie would see the light beam moving with speed c minus your 0.9c speed away from her, or 0.1c . . .*

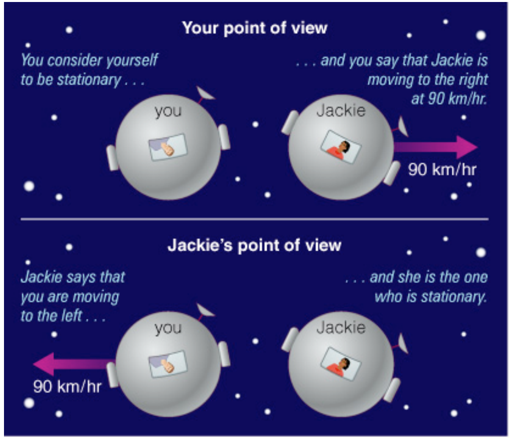
. . . but this is ***wrong,*** *because everyone measures the same speed of light.*



Jackie ***also*** *sees the light beam traveling at the full speed of light, c, despite your motion away from her.*

$c + 0.9c = c$!?!

Relativity of Motion



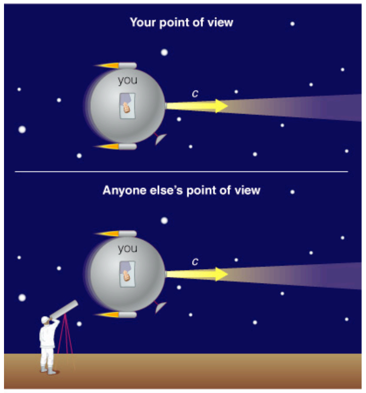
You consider yourself to be stationary . . .

. . . and you say that Jackie is moving to the right at 90 km/hr.

Jackie says that you are moving to the left . . .

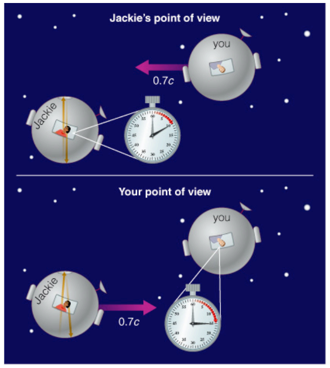
. . . and she is the one who is stationary.

Trying to Catch up to Light

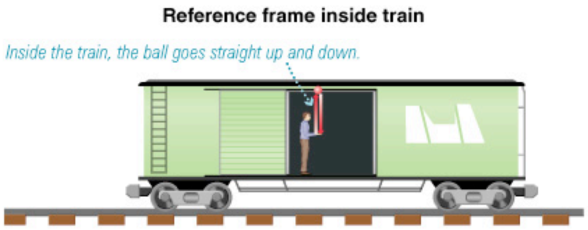


- Suppose you tried to catch up to your own headlight beams
- You'd always see them moving away at speed *c*
- Anyone else would also see the light moving ahead of you

How does relativity affect our view of time and space?



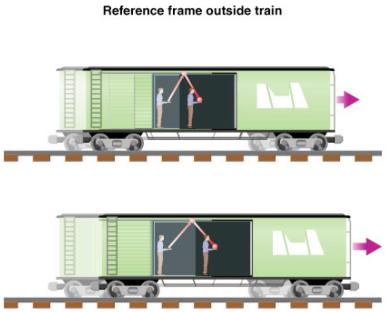
Path of Ball in a Stationary Train


Reference frame inside train

Inside the train, the ball goes straight up and down.

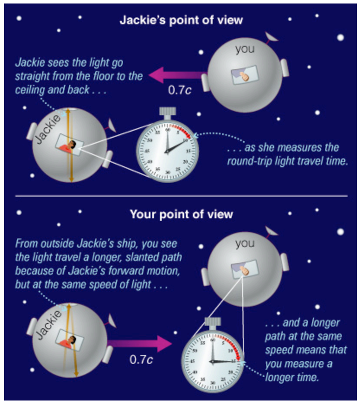
- Thinking about the motion of a ball on a train will prepare us for the next thought experiment

Path of Ball in a Moving Train


Reference frame outside train

- Someone outside the train would see the ball travel a longer path in one up-down cycle
- The faster the train is moving, the longer that path would be

Time Dilation


Jackie's point of view

Jackie sees the light go straight from the floor to the ceiling and back . . .

. . . as she measures the round-trip light travel time.

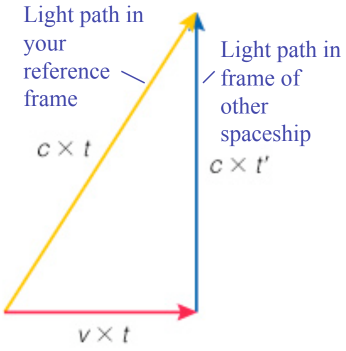
Your point of view

From outside Jackie's ship, you see the light travel a longer, slanted path because of Jackie's forward motion, but at the same speed of light . . .

. . . and a longer path at the same speed means that you measure a longer time.

- We can perform a thought experiment with a light beam replacing the ball
- The light beam, moving at *c*, travels a longer path in a moving object
- Time must be passing more slowly there

The Time Dilation Formula

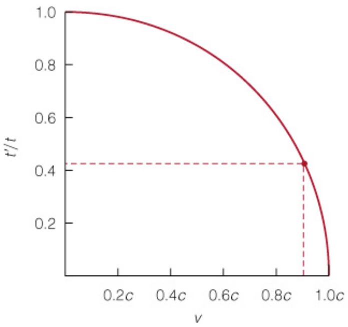


$$c^2t'^2 + v^2t^2 = c^2t^2$$

$$t'^2 = t^2 - \frac{v^2}{c^2}t^2$$

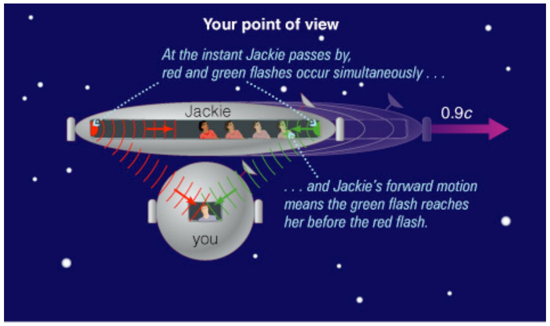
$$t' = t\sqrt{1-\left(\frac{v^2}{c^2}\right)}$$

The Time Dilation Formula



- Time will appear to pass more slowly in a moving object by an amount depending on its speed
- Time almost halts for objects nearing the speed of light

Simultaneous Events?

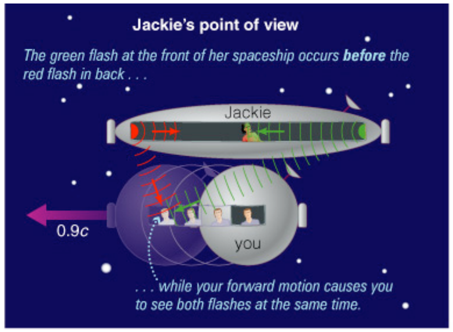


At the instant Jackie passes by, red and green flashes occur simultaneously . . .

. . . and Jackie's forward motion means the green flash reaches her before the red flash.

- In your reference frame, red and green lights on other spaceship appear to flash simultaneously

Simultaneous Events?

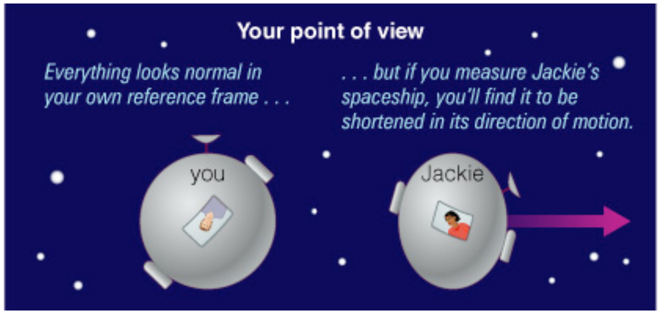


The green flash at the front of her spaceship occurs **before** the red flash in back . . .

. . . while your forward motion causes you to see both flashes at the same time.

- But someone on the other spaceship sees the green light flash first—*simultaneity is relative!*

Length Contraction



Everything looks normal in your own reference frame . . .

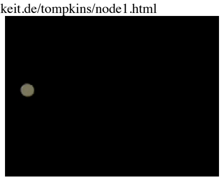
. . . but if you measure Jackie's spaceship, you'll find it to be shortened in its direction of motion.

- Similar thought experiments tell us that an object's length becomes shorter in its direction of motion

Length Contraction or Rotation?

http://www.tempolimit-lichtgeschwindigkeit.de/tompkins/node1.html

No relativity



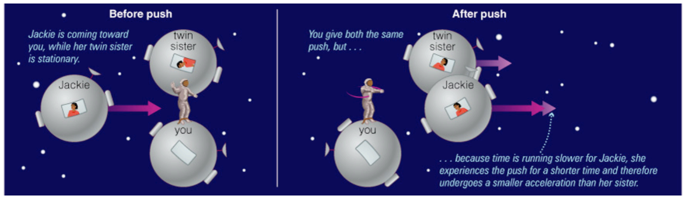
Length Contraction

Length Contraction+ Finite speed of light

Due to finite speed of light, object will be rotated.

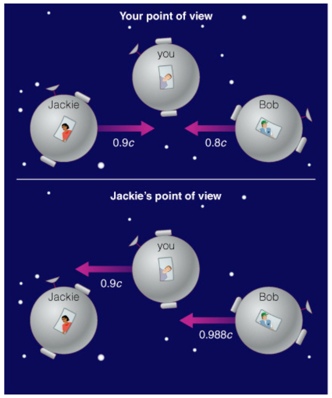
http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/ContractInvisible.html

Mass Increase



- A force applied to a rapidly moving object produces less acceleration than if the object were motionless
- This effect can be attributed to a mass increase in the moving object

Velocity Addition



Velocity of first ship in your frame $= v_1$

Velocity of second ship in frame of 1st $= v_2$

Velocity of second ship in your frame:

$$\frac{v_1 + v_2}{1 + \left(\frac{v_1}{c} \times \frac{v_2}{c}\right)}$$

Formulas of Special Relativity

Time Dilation: $t' = t\sqrt{1 - \left(\frac{v^2}{c^2}\right)}$

Length Contraction: $l' = l\sqrt{1 - \left(\frac{v^2}{c^2}\right)}$

Mass Increase: $m' = \frac{m}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$

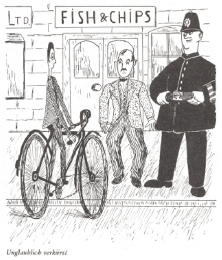
Deriving $E = mc^2$

$$m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \approx m_0\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) \qquad \text{for small } v$$

$$\text{Total energy } = mc^2 \approx \underbrace{m_0c^2}_{\text{Mass-Energy of object at rest}} + \underbrace{\frac{1}{2}m_0v^2}_{\text{Kinetic Energy}}$$

Why does this seem so strange?

In our world, v is much smaller than c



From Gamov's Mr. Thompson's Journey
(what if the speed of light was 30 km/hr)

Time Dilation: $t' = t\sqrt{1-\left(\frac{v^2}{c^2}\right)}$

Length Contraction: $l' = l\sqrt{1-\left(\frac{v^2}{c^2}\right)}$

Mass Increase: $m' = \frac{m}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}$

Since v/c is very small:

t' = t, l' = l, and m' = m

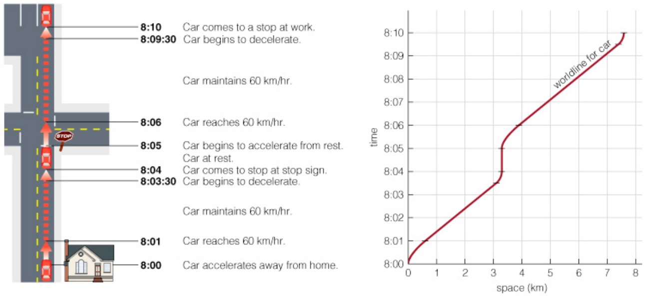
In our everyday experience

A Drive through Tuebingen

Every Reference Frame is Equal

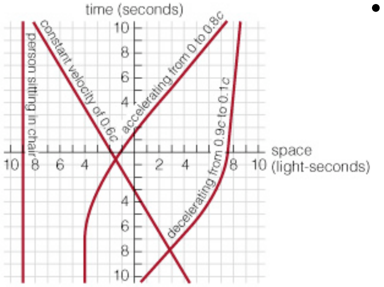
- According to you, time slows down in a moving spaceship
- According to someone on that spaceship, your time slows down
- Who is right?
- You both are, because time is not absolute but depends on your perspective

Spacetime Diagram of a Car



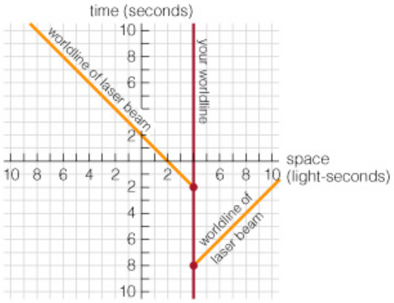
- A spacetime diagram plots an object's position in space at different moments in time

Worldlines



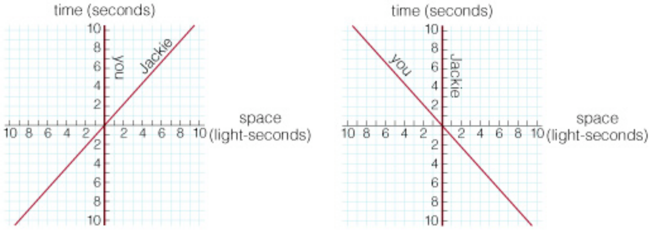
- A worldline shows an object's path through spacetime in a spacetime diagram
  - Vertical worldline: no motion
  - Diagonal worldline: constant-velocity motion
  - Curved wordline: accelerating motion

Worldlines for Light



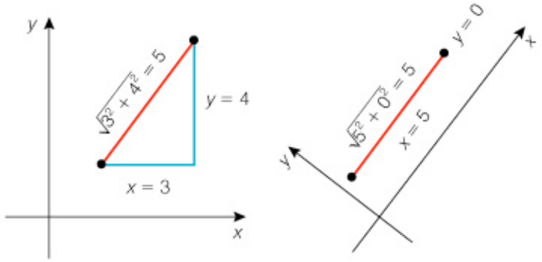
- Worldlines for light go at 45° angles in diagrams with light-seconds on one axis and seconds on the other

Worldlines and Relativity



- Worldlines look different in different reference frames

Worldlines and Relativity



Imagine two surveyors mapping a property. They both a different coordinate grid. They want to measure the distance between two trees.

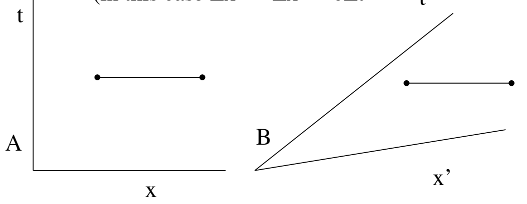
What is constant: the distance $x^2+y^2 = x'^2+y'^2$

What are not constant - x, y

Different Spacetime Coordinate Systems for Different Inertial Reference Frames

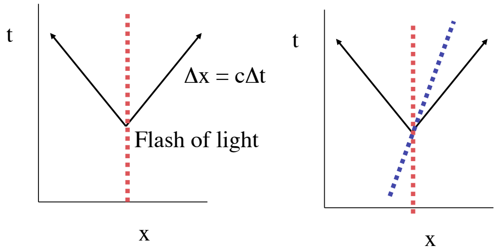
$$\Delta x^2 - c\Delta t^2 = \Delta x'^2 - c\Delta t'^2$$

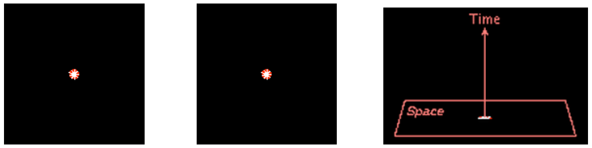
(in this case $\Delta x^2 = \Delta x'^2 - c\Delta t'^2$



- Two observers move past each other.
- Both have their own spacetime coordinate system
- They measure the same event.
- Observer A thinks the events are simultaneous
- Observe B does not.
- They can agree on one thing "interval" between two different events in spacetime: $x^2 + y^2 + z^2 - (ct)^2$

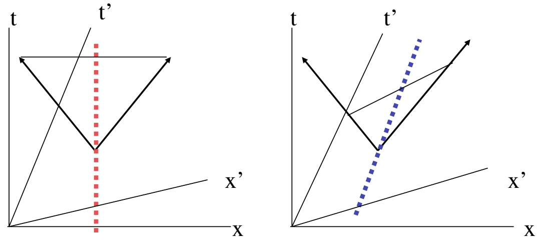
Light Cones

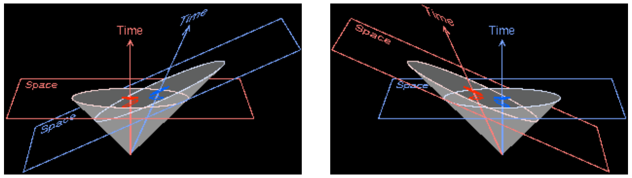




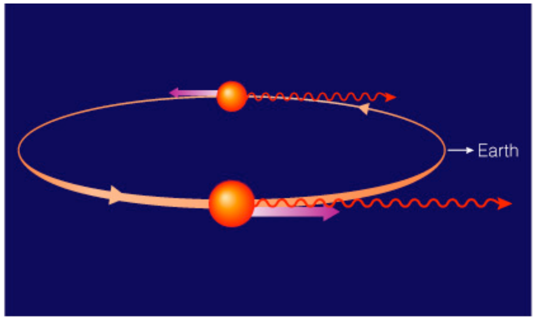
Andrew Hamilton http://casa.colorado.edu/~ajsh/sr/sr.shtml (excellent web site!!)

Solution





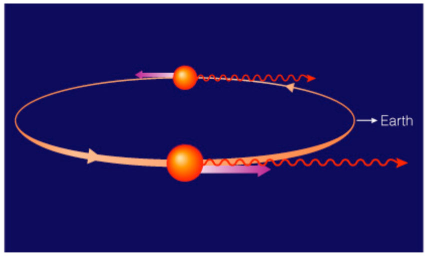
Do the effects predicted by relativity really occur?



Tests of Relativity

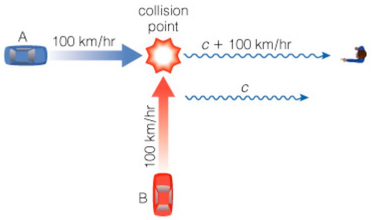
- First evidence for absoluteness of speed of light came from the *Michaelson-Morley Experiment* performed in 1887
- Time dilation happens routinely to subatomic particles the approach the speed of light in accelerators
- Time dilation has also been verified through precision measurements in airplanes moving at much slower speeds
- Prediction that $E=mc^2$ is verified daily in nuclear reactors and in the core of the Sun

Test Relativity for Yourself



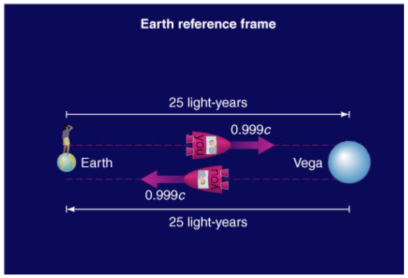
- If speed of light were not absolute, binary stars would not look like two distinct points of light
- You can verify relativity by simply looking through a telescope at a binary star system

A Paradox of Non-Relativistic Thinking



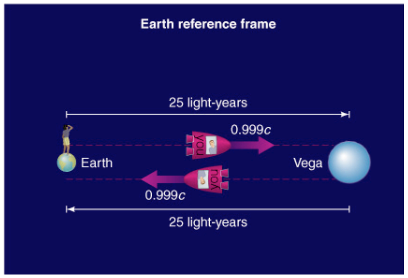
- If speed of light were not absolute, you would see the car coming toward you reach the collision point before the car it struck
- No paradox if light speed is same for everyone

A Journey to Vega



- The distance to Vega is about 25 light-years
- But if you could travel to Vega at 0.999*c*, the round trip would seem to take only two years!
- At that speed, the distance to Vega contracts to only 1 light-year in your reference frame
- Going even faster would make the trip seem even shorter!

A Journey to Vega



- However, your twin on Earth would have aged 50 years while you aged only 2
- There's a seeming contradiction to this conclusion:What does your twin see in his reference frame as he watches the Earth recede?

The Twin Paradox

http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/TwinParadox.html

The Twin Paradox

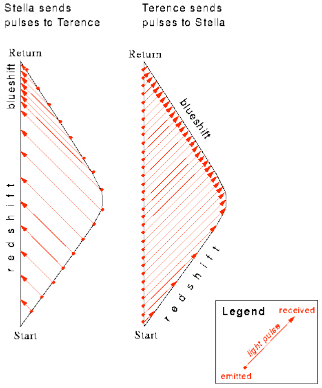


Figure 2: The Doppler Shift Explanation

The problem is solved when we realize that the twin does not stay in a single inertial reference frame.

He changes direction!

On his way back, he catches up with all the light pulses from Earth - everything speeds up!

What have we learned?

- How can we make sense of relativity?
  - We need abandon our old notions of space and time as absolute and adopt new a new common sense in which time and space depend on your perspective
  - We live in a low velocity world where the effects of relativity are not important.
  - We must consider spacetime as a single coordinate system (just like it doen’t make sense to have to consider just 1 dimension when we look at a map).
  - Physicist must use spacetime diagrams.

What have we learned?

- How does relativity affect our view of time and space?
  - Time slows down for moving objects
  - Lengths shorten for moving objects
  - Mass of a moving object increases
  - Simultaneity of events depends on your perspective
  - $E = mc^2$
- Do the effects predicted by relativity really occur?
  - Relativity has been confirmed by many different experiments
- How does special relativity offer us a ticket to the stars?
  - For someone moving near light speed, distances appear to become shorter because of length contraction

Relativity Web Sites

http://casa.colorado.edu/~ajsh/sr/sr.shtml Andrew Hamilton’s site

http://www.tempolimit-lichtgeschwindigkeit.de/ (mixed German and English)

http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/TwinParadox.html

http://faraday.physics.utoronto.ca/PVB/Harrison/SpecRel/Flash/ContractInvisible.html

http://www.upscale.utoronto.ca/PVB/Harrison/SpecRel/Flash/MichelsonMorley/MichelsonMorley.html

Special Topic: What if Light Can’t Catch You

- Is there a loophole?
- What if you’re somehow moving away from a distant planet faster than the speed of light?
- In that case you have no way of detecting that the planet is there.
- Although there are some phenomena that move faster than light, no *information* can be communicated faster than the speed of light