Standing waves

The interference of two sinusoidal waves of the same frequency and amplitude, travel in opposite direction, produce a standing wave.

\[ y_1(x, t) = y_m \sin(kx - \omega t), \quad y_2(x, t) = y_m \sin(kx + \omega t) \]

resultant wave: \[ y'(x, t) = [2y_m \sin kx] \cos(\omega t) \]
\[ y'(x, t) = [2y_m \sin kx] \cos(\omega t) \]

If \( kx = n\pi \) \((n = 0, 1, \ldots)\), we have \( y' = 0 \); these positions are called **nodes**. \( x = n\pi/k = n\pi/(2\pi/\lambda) = n(\lambda/2) \)
If \( kx = (n + \frac{1}{2})\pi \) \((n = 0, 1, \ldots)\), \( y' = 2y_m \) (maximum); these positions are called antinodes, \( x = (n + \frac{1}{2})(\lambda/2) \)
Standing waves and resonance

For a string clamped at both ends, at certain frequencies, the interference between the forward wave and the reflected wave produces a standing wave pattern. The string is said to resonate at these certain frequencies, called resonance frequencies.

$L = \lambda / 2, f = v / \lambda = v / 2L$  \hspace{1cm} 1\textsuperscript{st} harmonic or fundamental mode

$L = 2(\lambda / 2 ), f = 2(v / 2L)$  \hspace{1cm} 2\textsuperscript{nd} harmonic

$f = n(v / 2L), n = 1, 2, 3\ldots \hspace{1cm} n\textsuperscript{th} harmonic$
Reflection at a Boundary

Case a) String tied to wall.

–The reflected and incident pulses must have opposite signs. A node is generated at the end of the string.
Reflection at a Boundary

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Case b) String end can move.

- The reflected and incident pulses must have same sign. An antinode is generated at the end of the string.
Reflection at a Boundary

Case a) String tied to wall.
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Case b) String end can move.
- The reflected and incident pulses must have same sign. An antinode is generated at the end of the string.
A pulse is traveling down a wire and encounters a heavier wire. What happens to the reflected pulse?

1. The reflected wave changes phase
2. The reflected wave doesn’t change phase
3. There is no reflected wave
4. none of the above
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The speed of a traveling wave

\[ y(x, t) = y_m \sin(kx - \omega t + \phi) \]

Wave speed: \( v = \omega/k \)

since \( \omega = 2\pi/T, \quad k = 2\pi/\lambda \)

=> \( v = \lambda/T = \lambda f \)

Particle speed: \( u(x, t) = \frac{\partial y(x, t)}{\partial t} = -\omega y_m \cos(kx - \omega t + \phi) \)
Pipes

Open on both ends
Antinodes both ends.

Open on one end, closed on the other end

node on closed end, antinode on open end.
Chapter 17   Waves (Part II)

Sound wave is a longitudinal wave.
The speed of sound

Speed of a transverse wave on a stretched string

\[ v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertia property}}} \]
The speed of sound

Speed of a sound wave (longitudinal):

\[ v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertia property}}} \]

Speed of sound in gases & liquids is described by the bulk modulus \( B \) \((= - \Delta p/(\Delta V/V))\) and density \( \rho \):

\[ v = \sqrt{\frac{B}{\rho}} \]

Speed of sound:

air\((0^\circ C)\): 331 m/s, air\((20^\circ C)\): 343 m/s, water\((20^\circ C)\): 1482 m/s
The speed of sound

Speed of a sound wave (longitudinal):

\[ v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertia property}}} \]

Speed of sound in a solid is described by Young’s modulus \( E \) and density \( \rho \):

\[ v = \sqrt{\frac{E}{\rho}} \]

Speed of sound in aluminum: \( E = 7 \times 10^{10} \text{Pa} \), \( \rho = 2.7 \times 10^3 \text{kg/m}^3 \) \( \Rightarrow \) \( v = 5100 \text{ m/s} \),

In general, \( v_{\text{solid}} > v_{\text{liquid}} > v_{\text{gas}} \)
Traveling Sound Wave

To describe the sound wave, we use the displacement of an element at position \( x \) and time \( t \):

\[
s(x, t) = s_m \cos(kx - \omega t)
\]

- \( s_m \): displacement amplitude
- \( k = \frac{2\pi}{\lambda} \)
- \( \omega = \frac{2\pi}{T} = 2\pi f \)

As the wave moves, the air pressure at each point changes:

\[
\Delta p(x, t) = \Delta p_m \sin(kx - \omega t)
\]

Pressure amplitude: \( \Delta p_m = (v\rho\omega)s_m \)
Interference

Two sound waves traveling in the same direction, how they would interfere at a point P depends on the phase difference between them.

If the two waves were in phase when they were emitted, then the phase difference depends on path length difference: \( \Delta L = L_1 - L_2 \):

\[
\Delta \phi / 2\pi = \Delta L / \lambda
\]
Interference

For distant points,
\[ \Delta L = d \sin \theta \]
\[ \Delta \phi /2\pi = \Delta L/\lambda \]

If \( \Delta \phi = m \cdot (2\pi) \) or \( \Delta L/\lambda = m \) \((m = 0, 1, 2 \ldots)\)
We have fully constructive interference \( \Delta L = d \sin \theta = m\lambda \)

If \( \Delta \phi = (2m +1)\pi \) or \( \Delta L/\lambda = m + \frac{1}{2} \) \((m = 0, 1, 2 \ldots)\)
We have fully destructive interference \( d \sin \theta = (m + \frac{1}{2})\lambda \)

\[ y/D = \tan \theta \]
Today, a lightning bolt is seen in the distance. You start counting when you saw the flash. Twelve seconds elapsed until you heard the thunder. How far away was the lightning?

1) 4.1km  
2) 1200m  
3) 343m  
4) Yikes, head for cover  
5) none of the above
Checkpoint

Today, a lightning bolt is seen in the distance. You start counting when you saw the flash. Twelve seconds elapsed until you heard the thunder. How far away was the lightning?

1. 4.1km
2. 1200m
3. 343m
4. Yikes, head for cover
5. none of the above
Today, a lightning bolt is seen in the distance. You start counting when you saw the flash. Twelve seconds elapsed until you heard the thunder. How far away was the lightning?

\[ d = v_{\text{sound}} \cdot t = (343\text{m/s})(12\text{s}) \]
\[ = 4.1\text{km} = 2.5\text{miles} \]

1) 4.1km  
2) 1200m  
3) 343m  
4) Yikes, head for cover  
5) none of the above
**Intensity and Sound Level**

Intensity of a sound wave at a surface is defined as the average power per unit area

\[ I = \frac{P}{A} \]

For a wave \( s(x, t) = s_m \cos( kx - \omega t ) \)

\[ I = \frac{1}{2} \rho v \omega^2 s_m^2 \quad v = \frac{\omega}{k} \]

Variation of intensity with distance.

A point source with power \( P_s \)

\[ I = \frac{P_s}{4\pi r^2} \]

Spherical surface area: \( 4\pi r^2 \)
The figure indicates three small patches 1, 2, and 3 that lie on the surfaces of two imaginary spheres; the spheres are centered on an isotropic point source S of sound.

A) Rank the patches according to the intensity of the sound on them, greatest first.

\[ I = \frac{P_s}{4\pi r^2} \]

- \( r_1 = r_2 \Rightarrow I_1 = I_2 \)
- \( r_3 > r_1 \Rightarrow I_3 < I_1 \)

B) If the rates at which energy is transmitted through the three patches by the sound waves are equal, rank the patches according to their area, greatest first.

\[ I = \frac{P_s}{A} \]

- \( r_1 = r_2 \Rightarrow A_1 = A_2 \)
- \( r_3 > r_1 \Rightarrow A_3 > A_1 \)
The decibel Scale

The deci(bel) scale is named after Alexander Graham Bell.

Sound level $\beta$ is defined as

$$\beta = (10 \text{ dB}) \log (I/I_0)$$

Unit for $\beta$: dB (decibel)

$I_0$ is a standard reference intensity, $I_0 = 10^{-12} \text{ W/m}^2$

Threshold of hearing: $10^{-12}\text{W/m}^2$

Threshold of pain (depends on the type of music): $1.0\text{W/m}^2$

An increase of 10 dB => sound intensity multiplied by 10.
• The decibel scale
  – Sound intensity range for human ear: $10^{-12} - 1 \text{ W/m}^2$
  – More convenient to use logarithm for such a big range.
  – Sound level $\beta$ is defined as

$$\beta = (10 \text{ dB})\log_{10}(I/I_0)$$

• Unit for $\beta$: dB (decibel)

$I_0$ is a standard reference intensity, $I_0 = 10^{-12}$ W/m$^2$

Examples:
  at hearing threshold: $I = I_0, \quad \beta = 10 \log_{10} 1 = 0$,
  normal conversation: $I \sim 10^6 I_0, \quad \beta = 10 \log_{10} 10^6 = 60 \text{ dB}$
  Rock concert: $I = 0.1 \text{ W/m}^2, \quad \beta = 10 \log_{10}(0.1/10^{-12}) = 110 \text{ dB}$
Beats

Two sound waves with frequencies $f_1$ and $f_2$ ($f_1 \sim f_2$) reach a detector, the intensity of the combined sound wave vary at beat frequency $f_b$

$$f_b = f_1 - f_2$$
The Doppler effect

When the source or the detector is moving relative to the media, a frequency which is different from the emitted frequency is detected.

This is called **Doppler effect**.

**Case 1**: source stationary, the detector is moving towards the source:

The detected frequency is:

\[ f' = f \frac{v + v_D}{v} \]

Note: \( f' \) is *greater* than \( f \).
Case 2: source stationary, detector moving away from the source:
\[ f' = f \frac{v - v_D}{v} \]

\( f' \) is \textit{smaller} than \( f \).

Cases 3 & 4: source moving, detector stationary
\[ f' = f \frac{v}{v \pm v_s} \]

Source is moving toward detector, “−” sign, \( f' > f \)
Source is moving away from detector, “+” sign, \( f' < f \)
General formula for Doppler effect

\[ f' = f \frac{v \pm v_D}{v \pm v_S} \]

v: speed of sound  
v_D: detector speed  

v_S: source speed

Rules:

Moving *towards* each other: frequency *increases*

Moving *away from* each other: frequency *decreases*

For numerator,

if detector is moving *towards* the source, “+”

if detector is moving *away from* the source, “−”

For denominator,

if source is moving *away from* the detector, “+”

if source is moving *towards* the detector, “−”