Chapter 12 Equilibrium & Elasticity

If there is a **net force**, an object will experience a linear acceleration. *(period, ...end of story!)*

If there is a **net torque**, an object will experience an angular acceleration. *(period, ...end of story!)*

How then can we keep things from moving?

Recall, \( \vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \) and \( \vec{\tau} = \frac{d\vec{L}}{dt} \)
Chapter 12 Equilibrium & Elasticity

- Equilibrium: $\dot{P} = \text{constant}$ and $\dot{L} = \text{constant}$
- Static equilibrium: Objects that are not moving either in translation or rotation. $\dot{P} = 0 \quad \dot{L} = 0$
- Requirements of equilibrium
  \[ \vec{F}_{\text{net}} = \frac{d\dot{P}}{dt} \quad \vec{F}_{\text{net}} = 0 \quad (\text{balance of forces}) \]

  \[ \vec{\tau} = \frac{d\dot{L}}{dt} \quad (\text{general torque balance}) \quad 0 = \int_{\text{net}} \vec{\tau} \]
Consider a situation in which the forces that act on the body lie only in the xy plane.

\[
\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} = F_x \hat{i} + F_y \hat{j} = m\ddot{a}
\]

We *must* have \( F_{\text{net,x}} = 0 \) and \( F_{\text{net,y}} = 0 \) or else it *will* accelerate in the x-y plane.

The only torque that can act on the body is \( \tau_z \) since \( \tau \) has to be *perpendicular* to the forces (in the x-y plane).

\( \tau_{\text{net,z}} \) is the net torque that the external forces produce either about the z axis or about any axis parallel to it

\( \tau_{\text{net,z}} \) *must* be \( = 0 \) or else it *will* rotate.
Center of gravity

**Center of gravity**: gravitational force on a body effectively act at a single point.

For everyday objects, the center of gravity is coincident with its *center of mass*. 
Sample Problem 12-1

A beam of length L and mass \( m = 1.8 \text{ kg} \), is at rest with its ends on two scales. A block of \( M = 2.7 \text{ kg} \), is at rest on the beam, with its center a distance \( L/4 \) from the beam’s left end. What do the scales read (\( F_L \) and \( F_R \))?
Sample Problem 12-1

A beam of length $L$ and mass $m = 1.8$ kg, is at rest with its ends on two scales. A block of $M = 2.7$ kg, is at rest on the beam, with its center a distance $L/4$ from the beam’s left end. What do the scales read?

Sum of all the forces on the beam must equal zero.

\[
\hat{y} : \sum_i \vec{F}_i = 0 \Rightarrow F_l - Mg - mg + F_r = 0
\]
Sample Problem 12-1

A beam of length \( L \) and mass \( m = 1.8 \) kg, is at rest with its ends on two scales. A block of \( M = 2.7 \) kg, is at rest on the beam, with its center a distance \( L/4 \) from the beam’s left end. What do the scales read?

Sum of all the torques about any point on the beam must equal zero. Choose this point to be the left end of the beam.

\[
\hat{z} : \sum \vec{\tau}_i = 0 \implies (0)F_l - \left(\frac{L}{4}\right)Mg - \left(\frac{L}{2}\right)mg + (L)F_r = 0
\]
Sample Problem 12-1

A beam of length $L$ and mass $m = 1.8$ kg, is at rest with its ends on two scales. A block of $M = 2.7$ kg, is at rest on the beam, with its center a distance $L/4$ from the beam’s left end. What do the scales read?

Which yields,

$$F_r = \left(\frac{L}{4}\right) Mg + \left(\frac{L}{2}\right) mg$$

and

$$F_l = Mg + mg - F_r = \left(\frac{3L}{4}\right) Mg + \left(\frac{L}{2}\right) mg$$
Checkpoint 2

What should $F_1$ be in order to keep the uniform rod in static equilibrium?
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What should $F_1$ be in order to keep the uniform rod in static equilibrium?

\[ F_1 = \]

1. 0.0 N
2. 40 N
3. 45 N
4. it’s pointed in the wrong direction
5. none of the above
Checkpoint 2

\[ \sum_i \vec{F}_i = 0 \Rightarrow 20 \, \text{N} - 10 \, \text{N} - F_1 - 30 \, \text{N} + 65 \, \text{N} = 0 \Rightarrow F_1 = 45 \, \text{N} \]

\[ \sum_i \tau_i = 0 \]

\[ \Rightarrow -(8d)20 \, \text{N} + (4d)10 \, \text{N} + (2d)F_1 + (d)30 \, \text{N} - (0d)65 \, \text{N} = 0 \Rightarrow F_1 = 45 \, \text{N} \]

What should \( F_1 \) be in order to keep the uniform rod in static equilibrium?

1) 0 N  
2) 40 N  
3) 45 N  
4) it’s pointed in the wrong direction  
5) none of the above
Stress and Strain

Microscopic view of materials: Materials are made of atoms held in place by electrostatic interactions with neighboring atoms.

These interactions are such that the atoms are constantly in harmonic motion about their equilibrium positions.
Stress and Strain

External forces can be exerted on these atoms. The atoms will react to these forces depending on their microscopic environment.
Deformation Types

**elargonation (tensile strength)**  

**shear**

**hydraulic compression**

modulus is a constant depended on how much the materials react (deform) to the applied forces.

\[ \text{stress} = \text{modulus} \times \text{strain} \]
Elongation (tensile strength) and compression

Stress is defined as perpendicular force per unit area. **Stress** = \( \frac{F}{A} \)

Strain is defined as the fractional change in the length of the object. **Strain** = \( \frac{\Delta L}{L} \)

**Young’s modulus** \( E \) is the proportionality constant.

\[
\frac{F}{A} = E \frac{\Delta L}{L}
\]
**Shear stress**

Stress is defined as force in the plane per unit area. \( \text{Stress} = \frac{F}{A} \)

Strain is defined as the fractional change in the movement of the object.
\( \text{Strain} = \frac{\Delta x}{L} \)

**Shear modulus** \( G \) is the proportionality constant.

\[
\frac{F}{A} = G \frac{\Delta x}{L}
\]
Hydraulic Stress

For three dimensions we use pressure, which is also defined as force per unit area. **Pressure** \( p = \frac{F}{A} \)

Strain is defined as the fractional change in the volume of the object. **Strain** = \( \frac{\Delta V}{V} \)

**Bulk modulus** \( B \) is the proportionality constant.

\[
p = \frac{F}{A} = B \frac{\Delta V}{V}
\]
Materials science studies cause of specific shape of this curve.

Region we’ll be discussing this semester.

Additional region of interest to engineers.
Study Suggests Design Flaws Didn't Doom Towers
By ERIC LIPTON

NY Times Article
Published: October 20, 2004


This article discusses the stresses, strains exerted on the supporting columns.