A uniform sphere of mass \( m = 0.85 \text{ kg} \) and radius \( r = 4.2 \text{ cm} \) is held in place by a massless rope attached to a frictionless wall a distance \( L = 8.0 \text{ cm} \) above the center of the sphere. Find (a) the tension in the rope and (b) the force on the sphere from the wall.

**Three forces:**
- Tension along the rope
- \( F_N \) from wall
- \( mg \) acting down

\[
\begin{align*}
\text{(1) Vertical:} & \quad T \cos \theta - mg = 0 \\
\text{(2) Horizontal:} & \quad F_N - T \sin \theta = 0
\end{align*}
\]

**(a) Solve (1) for \( T \):**

\[
T = \frac{mg}{\cos \theta} \quad \text{but} \quad \cos \theta = \frac{L}{\sqrt{L^2 + r^2}}
\]

\[
T = \frac{mg \sqrt{L^2 + r^2}}{L} = \frac{(0.85 \text{ kg})(9.8 \text{ m/s}^2)(4.2 \text{ cm})}{0.08 \text{ m}}
\]

\[
T = 9.4 \text{ N}
\]

**(b) Solve (2) for \( F_N \):**

\[
F_N = T \sin \theta \quad \text{but} \quad \sin \theta = \frac{r}{\sqrt{L^2 + r^2}}
\]

\[
F_N = \frac{mg \sqrt{L^2 + r^2}}{L} \cdot \frac{r}{\sqrt{L^2 + r^2}} = \frac{mgr}{L}
\]

\[
F_N = \frac{(0.85 \text{ kg})(9.8 \text{ m/s}^2)(4.2 \text{ cm})}{0.08 \text{ m}}
\]

\[
F_N = 4.4 \text{ N}
\]
16. A man is trying to get his car out of mud. He ties one end of a rope tightly around the front bumper and the other end tightly around a utility pole 18 m away. He then pushes sideways on the rope at its midpoint with a force of 550 N, displacing the center of the rope 0.3 m from its previous position and the car barely moves. What is the magnitude of the force on the car from the rope? (The rope stretches somewhat.)

![Diagram of the situation]

So what we need to find is the tension in the rope.

If we find the angle, $\theta$, that the rope makes with the car with respect to the dashed line

$$\theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{0.30 \text{ m}}{9.0 \text{ m}} \right)$$

$$\theta = 1.9^\circ$$

Then, where $\vec{F}$ is applied

$$T_x = \sin \theta = \frac{F}{\frac{F}{\sin \theta}}$$

but, we need the tension from both "sides" of the rope, so we must multiply by 2.

$$T_x = \frac{F}{2 \sin \theta} = \frac{550 \text{ N}}{2 \sin(1.9^\circ)} = 8.3 \times 10^3 \text{ N}$$
Chapter 12

12. The system shown is in equilibrium with the string in the center exactly horizontal. Block A weighs 40 N, block B weighs 50 N, and angle $\theta = 35^\circ$.

Find (a) tension $T_1$, (b) tension $T_2$, (c) tension $T_3$, and (d) angle $\theta$.

Looking at block A

Vertical: $T_1 \cos \theta - m_A g = 0$
Horizontal: $T_1 \sin \theta = T_2$

(a) $T_1 = \frac{m_A g}{\cos \theta} = \frac{40 N}{\cos (35^\circ)} = 49 N$

(b) $T_2 = ?$

$T_2 = T_1 \sin \theta = (49 N) \sin 35^\circ = 28 N$

(c) For block B

Vertical: $T_2 \cos \theta - m_B g = 0$
Horizontal: $T_2 - T_3 \sin \theta = 0$

Component of $T_3$ in $x$ direction = $T_{3x}$
Component of $T_3$ in $y$ direction = $T_{3y}

So $T_3 = \sqrt{T_{3x}^2 + T_{3y}^2} = \sqrt{(28 N)^2 + (50 N)^2} = 57 N$
12. (d) \[ \Theta = \tan^{-1} \left( \frac{T \sin \alpha}{T_{3x}} \right) = \tan^{-1} \left( \frac{28}{50} \right) \]

\[ \Theta = 29^\circ \]

2. 7. The figure shown is in equilibrium.
A concrete block of mass 22.5 Kg hangs from the end of the uniform strut of mass 45.0 Kg. For angles \( \Phi = 30.0^\circ \) and \( \Theta = 45.0^\circ \), find:
(a) the tension \( T \), and
(b) the horizontal and vertical components of the force on the strut from the hinge.

The angle \( \alpha \) is given by \( \alpha = \Theta - \Phi = 45^\circ - 30^\circ = 15^\circ \)

\[ \beta = 90^\circ - 45^\circ \]

Define angle \( \alpha = \) angle between cable and strut
angle \( \beta = \) angle between strut and vertical force (weight)
\( M = 225 \text{ Kg} \);
\( m = 45.0 \text{ Kg} \)

\( l = \) length of strut

Compute torques about the hinge:

\[ T \sin \alpha = Mg l \sin \beta + Mg \left( \frac{L}{2} \right) \sin \beta \]

\[ T = \frac{Mg \sin \beta + \frac{1}{2} Mg \sin 15^\circ}{\sin \alpha} \]

\( = \frac{(225)(9.8 \text{ m/s}^2) \sin 45^\circ + \frac{1}{2}(45 \text{ Kg}) (9.8 \text{ m/s}^2) \sin 45^\circ}{\sin (15^\circ)} \)
(b) horizontal hinge force must be

\[ F_x = T \cos \theta \]
\[ = \left( 6.63 \times 10^3 \text{N} \right) \cos (30^\circ) \]
\[ F_x = 5.74 \times 10^3 \text{N} \]

(c) vertical hinge force must be

\[ F_y = T \sin \theta + Mg + mg \]
\[ = 6.63 \times 10^3 \text{N} + (22\text{kg})(9.8 \text{m/s}^2) + (45\text{kg})(9.8 \text{m/s}^2) \]
\[ F_y = 5.96 \times 10^3 \text{N} \]
A 103-Kg uniform log hangs by two steel wires, A and B, both of radius 1.20 mm. Initially, wire A was 2.50 m long and 2.00 mm shorter than wire B. The log is now horizontal.

What are the magnitudes of the forces on it from
(a) wire A and (b) wire B? (c) What is the ratio \( \frac{F_A}{F_B} \)?

![Diagram](image)

\( F_A \) is force on log by wire A; \( L_A \) is original length of wire A.
\( F_B \) is force on log by wire B; \( L_B \) is original length of wire B.
\( m \) is mass of log
\( \Delta L_A \) is change in length of wire A
\( \Delta L_B \) is change in length of wire B
\( A \) is cross-sectional area of wire
\( E \) is Young's modulus for wire (for steel: \( E = 200 \times 10^9 \text{ N/m}^2 \))

Since the log is in static equilibrium, the sum of the forces must be zero.

\[
F_A + F_B - mg = 0
\]

\[
\Delta L_A = \frac{F_A L_A}{AE}
\]

\[
\Delta L_B = \frac{F_B L_B}{AE}
\]

\( \Rightarrow \frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + l \)

Solve for \( F_B \):
\[
F_B = \frac{F_A L_A}{L_B} - \frac{AE l}{L_B}
\]

Then substitute into original expression and solve for \( F_A \).
45. (continued)  \[ F_A = \frac{mgL_b + AE \ell}{L_a + L_b} \]

We know all quantities except area, so:

\[ A = \pi r^2 = \pi (1.20 \times 10^{-3} \text{ m})^2 = 4.52 \times 10^{-6} \text{ m}^2 \]

For simplicity, we can assume that \( L_A = 2.50 \text{ m} \) and \( L_B = 2.50 \text{ m} \):

\[
F_A = \frac{(103 \text{ kg})(9.8 \text{ m/s}^2)(2.50 \text{ m}) + (4.52 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ N/m}^2)(2.0 \times 10^{-3} \text{ m})}{2.50 \text{ m} + 2.50 \text{ m}}
\]

\[ F_A = 866 \text{ N} \]

(b) \[ F_B = mg - F_A = (103 \text{ kg})(9.8 \text{ m/s}^2) - 866 \text{ N} \]

\[ F_B = 143 \text{ N} \]

(c) The sum of the torques must be zero, so:

\[ F_A d_A - F_B d_B = 0 \]

\[ \frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143 \text{ N}}{866 \text{ N}} \]

\[ \frac{d_A}{d_B} = 0.165 \]