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Branching Fractions and Transition Probabilities for Ga II, In II and Tl II from Measured Lifetime and Energy Level Data

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Abstract

Branching fractions and relative transition probabilities are predicted for the ns^2-nsnp and nsnp-nsnd supermultiplets in Ga II, In II and Tl II through a data-based systematization of measured lifetime and spectroscopic energy level data. A formalism is used that parametrizes the effects of Coulomb exchange, spin-orbit and spin-other-orbit interactions, and differences between the radial wave functions of the singlet and triplet states to deduce effective intermediate coupling amplitudes. Connections between singlet and triplet lifetimes and among triplet lifetimes are examined and predictions are made where data do not currently exist.

1. Introduction

A knowledge of the relative transition probability rates within a supermultiplet transition array has applications in many areas. In emission studies this information can be used either to determine the relative populations of the upper levels from measured relative intensities, or to predict emitted intensities from a population model. If the multiplet is the only exit channel for the decay of one or more of the upper levels, the absolute transition probability can be determined from measurements of the lifetime of the level and the relative branching fractions of the corresponding subset within the transition array. Knowing the absolute transition probabilities, these can be converted to oscillator strengths to deduce elemental abundances from absorption spectra.

While many measurements have been made of lifetimes in atoms and ions and of branching fractions for transitions in the visible region for neutral atoms, very little information is currently available concerning branching fractions for charged ions or for transitions in the ultraviolet region [1]. The reasons for this lack of data are clear, and involve special difficulties associated with the relative calibration of detection systems for use with ions and ultraviolet emission.

However, semiempirical methods exist for treating divalent systems that utilize singlet-triplet intermediate coupling amplitudes obtained from spectroscopic energy levels to predict relative transition probability rates between and among polyads within a supermultiplet [2–9]. While this method requires that both the upper and lower configuration state vectors be dominated by a single configuration, the formulation can effectively characterize the effects of spin-own-orbit, spin-other-orbit, and indirect configuration interaction effects that induce differences between the singlet and triplet radial wave functions.

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These methods have been quite successfully applied to the ground state transitions in neutral Zn I, Cd I, and Hg I atoms [6–9]. A study is reported here that codifies this method, extends its application to ions and to transitions between excited states (in which both the upper and lower states possess singlet-triplet mixing), and combines the results with existing lifetime measurements to obtain absolute atomic transition probabilities. The ions studied were Ga II, In II and Tl II (isoelectronic to Zn I, Cd I and Hg I). The study not only yields new atomic structure data for these ions, but also provides a simple and readily applicable method that can be used in combination with precision ionic lifetime measurements to yield oscillator strength data.

2. Calculational formalism

The $nsn'\ell$ configuration consists of four levels which, in the limit of pure LS coupling, can be denoted by the standard spectroscopic symbols ${}^3L_{\ell+1}, {}^3L_\ell, {}^1L_\ell, {}^3L_{\ell-1}$. Under the conditions of intermediate coupling, the physical $J=1$ levels can be described by the wave functions

$$|{}^3L'_\ell> = |{}^3L_\ell> \cos\theta - |{}^1L_\ell> \sin\theta \tag{1}$$

$$|{}^1L'_\ell> = |{}^3L_\ell> \sin\theta + |{}^1L_\ell> \cos\theta \tag{2}$$

where θ is the singlet-triplet mixing angle. The mixing can be expressed in terms of the Slater exchange energy parameter G_1 and the diagonal and off-diagonal magnetic energy parameters μ_1 and μ_2 as [3,5]

$$\cot(2\theta) = \frac{2G_1 + \mu_1/2}{\sqrt{\ell(\ell+1)}\mu_2}. \tag{3}$$

In the simplest formulation [2,3], the diagonal and off-diagonal magnetic parameters are both set equal to the standard spin-orbit energy yielding $\zeta = \mu_1 = \mu_2$. However, already in 1932 Wolfe [4] showed that this formulation can be extended to include the spin-other-orbit interaction energy η. (A lucid exposition of this formulation is given in Ref. [5]). This correction contributes constructively to the diagonal and destructively to the off-diagonal matrix elements, yielding $\mu_1 = \zeta + \eta$ and $\mu_2 = \zeta$-η. The same effective parametrization with a different phenomenological origin was independently suggested in 1939 by King and VanVleck [6], who argued that the radial wave functions may be slightly different for singlet and triplet states within the same configuration, and hence yield different diagonal and off-diagonal spin-orbit integrals. Thus this parametrization yields $\mu_1 = \zeta_{33}$ denoting the triplet-triplet integral and

$\mu_2 = \zeta_{13}$ denoting the singlet-triplet integral. A detailed description of these methods with source references and applications to the Hg I system has been given by Benck *et al.* [8]. Vainshtein and Poluektov [9] have made an extension of the method to also include spin-spin interaction, but their development shows that this inclusion increases the number of independent parameters to four, precluding a semiempirical determination from the three measured energy level splittings without additional assumptions.

Since both of the physical mechanisms described above lead to the same expression as given by Eq. (3), the determination of these parameters from measured spectroscopic data can effectively account for both spin-other-orbit interaction and differences between the singlet and triplet radial wave functions. Such differences could arise from configuration interaction (CI), and this is particularly important for *nsn*d levels, since CI with $n\mathrm{p}^2$ primarily affects the $^1\mathrm{D}_2$ level.

The physical levels are specified by the Slater parameters through the relationships [5]

$$^3\mathrm{L}_{\ell+1} = E_0 - G_1 + \ell\mu_1/2, \tag{4}$$

$$^1\mathrm{L}'_\ell, ^3\mathrm{L}'_\ell = E_0 - \mu_1/4 \pm \Delta, \tag{5}$$

$$^3\mathrm{L}_{\ell-1} = E_0 - G_1 - (\ell+1)\mu_1/2, \tag{6}$$

$$\Delta = \sqrt{(G_1 + \mu_1/4)^2 + \ell(\ell+1)\mu_2^2/4}. \tag{7}$$

The three parameters G_1, μ_1, μ_2 are merely a one-to-one re-characterization of the measured spectroscopic data, since they are uniquely specified by the measured energy level data for the three independent level splittings ($^1\mathrm{L}'_\ell - ^3\mathrm{L}_{\ell+1}$), ($^3\mathrm{L}'_\ell - ^3\mathrm{L}_{\ell-1}$), ($^3\mathrm{L}_{\ell+1} - ^3\mathrm{L}_{\ell-1}$) of the four levels (with the measured energy levels denoted here by their spectroscopic symbols). From Eqs. (4–7) it can be seen that the remapping can be achieved using the relationships

$$G_1 = (^1\mathrm{L}'_\ell - ^3\mathrm{L}_{\ell+1} + ^3\mathrm{L}'_\ell - ^3\mathrm{L}_{\ell-1})/2, \tag{8}$$

$$\mu_1 = 2(^3\mathrm{L}_{\ell+1} - ^3\mathrm{L}_{\ell-1})/(2\ell+1) \tag{9}$$

with these results then used to specify

$$\mu_2 = \left[\frac{(^1\mathrm{L}'_\ell - ^3\mathrm{L}'_\ell)^2 - (2G_1 + \mu_1/2)^2}{\ell(\ell+1)}\right]^{1/2}. \tag{10}$$

If spin-other-orbit and singlet-triplet radial wave function differences are neglected, the fitting parameters are overdetermined and Eq. (10) can be used as a test of the single configuration picture. If all three parameters are fitted the overdetermination is removed, but it is possible to test the validity of the single configuration model by comparisons with measured *g*-factor and transition probability data.

Here the formalism will be applied to supermultiplets connecting the s^2 and sp configurations as well as the sp and sd configurations. The *LS* basis states will be denoted by their corresponding spectroscopic symbols, and the physical wave functions will be denoted by the same symbols with primes attached to indicate the presence of singlet-triplet mixing. For s^2 there is only one state and it is pure singlet, $|^1\mathrm{S}_0>$. For sp there are the two pure states $|^3\mathrm{P}_2>$ and $|^3\mathrm{P}_0>$, and the two mixed states

$$|^3\mathrm{P}'_1> = |^3\mathrm{P}_1>\cos\theta_1 - |^1\mathrm{P}_1>\sin\theta_1, \tag{11}$$

$$|^1\mathrm{P}'_1> = |^3\mathrm{P}_1>\sin\theta_1 + |^1\mathrm{P}_1>\cos\theta_1. \tag{12}$$

For sd there are the two pure states $|^3\mathrm{D}_3>$ and $|^3\mathrm{D}_1>$ and the two mixed states

$$|^3\mathrm{D}'_2> = |^3\mathrm{D}_2>\cos\theta_2 - |^1\mathrm{D}_2>\sin\theta_2, \tag{13}$$

$$|^1\mathrm{D}'_2> = |^3\mathrm{D}_2>\sin\theta_2 + |^1\mathrm{D}_2>\cos\theta_2. \tag{14}$$

The transition moments among these wave functions can be evaluated in terms of the mixing angles and the transition moments in pure spin-orbit coupling, which are given by [10]

$$<^{2S+1}P_J|r|^{2S+1}D_{J'}> = (-)^{J'+S}\sqrt{(2J+1)(2J'+1)} \times \begin{Bmatrix} 1 & S & J \\ J' & 1 & 2 \end{Bmatrix} \mathcal{M}_{\ell\ell'} \tag{15}$$

where $\mathcal{M}_{\ell\ell'}$ is the interconfigurational radial matrix element. In the non relativistic Schrödinger approximation $\mathcal{M}_{\ell\ell'}$ is the same for all members of the supermultiplet. In calculations of emission branching fractions $\mathcal{M}_{\ell\ell'}$ is often assumed to be the same for all transitions from the same upper level, whereas in calculations of absorption branching fractions, it is similarly assumed to be the same for all transitions from the same lower level. In cases in which the transition moment is not strongly influenced by cancellation effects or CI, its variation over the multiplet is often negligible.

The transition moments can be formed from Eqs. (2 –15). For the s^2-sp transitions these are

$$<^1\mathrm{S}_0|\mathrm{r}|^3\mathrm{P}_1> = [\sin\theta_1]\mathcal{M}_{\mathrm{sp}} \tag{16}$$

$$<^1\mathrm{S}_0|\mathrm{r}|^1\mathrm{P}_1> = [\cos\theta_1]\mathcal{M}_{\mathrm{sp}} \tag{17}$$

and for sp-sd

$$<^3\mathrm{P}_0|\mathrm{r}|^3\mathrm{D}_1> = \left[\frac{1}{\sqrt{3}}\right]\mathcal{M}_{\mathrm{pd}}, \tag{18}$$

$$<^3\mathrm{P}'_1|\mathrm{r}|^3\mathrm{D}_1> = \left[\frac{1}{2}\cos\theta_1\right]\mathcal{M}_{\mathrm{pd}}, \tag{19}$$

$$<^3\mathrm{P}_2|\mathrm{r}|^3\mathrm{D}_1> = \left[\frac{1}{\sqrt{60}}\right]\mathcal{M}_{\mathrm{pd}}, \tag{20}$$

$$<^1\mathrm{P}'_1|\mathrm{r}|^3\mathrm{D}_1> = \left[\frac{1}{2}\sin\theta_1\right]\mathcal{M}_{\mathrm{pd}}, \tag{21}$$

$$<^3\mathrm{P}'_1|\mathrm{r}|^3\mathrm{D}'_2> = \left[\frac{\sqrt{3}}{2}\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\right]\mathcal{M}_{\mathrm{pd}}, \tag{22}$$

$$<^3\mathrm{P}_2|\mathrm{r}|^3\mathrm{D}'_2> = \left[\frac{1}{2}\cos\theta_2\right]\mathcal{M}_{\mathrm{pd}}, \tag{23}$$

$$<^1\mathrm{P}'_1|\mathrm{r}|^3\mathrm{D}'_2> = \left[\frac{\sqrt{3}}{2}\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2\right]\mathcal{M}_{\mathrm{pd}}, \tag{24}$$

$$<^3\mathrm{P}_2|\mathrm{r}|^3\mathrm{D}_3> = \left[\sqrt{\frac{7}{5}}\right]\mathcal{M}_{\mathrm{pd}}, \tag{25}$$

$$<^3\mathrm{P}'_1|\mathrm{r}|^1\mathrm{D}'_2> = \left[\frac{\sqrt{3}}{2}\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2\right]\mathcal{M}_{\mathrm{pd}}, \tag{26}$$

$$<^3\mathrm{P}_2|\mathrm{r}|^1\mathrm{D}'_2> = \left[\frac{1}{2}\sin\theta_2\right]\mathcal{M}_{\mathrm{pd}}, \tag{27}$$

$$<^1\mathrm{P}'_1|\mathrm{r}|^1\mathrm{D}'_2> = \left[\frac{\sqrt{3}}{2}\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2\right]\mathcal{M}_{\mathrm{pd}}. \tag{28}$$

The transition probabilities can then be written as

$$g_u A_{ul}(\mathrm{ns}^{-1}) = \left[\frac{1265.38}{\lambda(\text{Å})}\right]^3 |< u|r|l >|^2. \quad (29)$$

The relative transition probabilities for the supermultiplet fractions R_{ik} can be defined

$$R_{ul} \equiv A_{ul} / \sum_{u'l'} A_{u'l'}. \quad (30)$$

Under the assumption that $\mathcal{M}_{\ell\ell'}$ is the same for all transitions in the supermultiplet it cancels in the ratio of Eq. (30.) In this approximation it is possible to predict from R_{ik} both the ratios of the lifetimes of the various upper levels and the branching fractions for the various decay branches of each upper level. If C_i is the sum over lower levels for the upper level i, this quantity should be proportional to the reciprocal lifetime

$$C_i = \sum_k R_{ik} \propto 1/\tau_i. \quad (31)$$

By comparing the triplet-to-triplet and singlet-to-triplet values of $C_i\tau_i$ it is possible to test the assumptions of this method and to make predictions where data are not available. The branching fractions can be obtained from $BF_{ik} \equiv R_{ik}/C_i$, and used to predict transition probabilities where lifetime measurements are available.

In this formulation there are two distinct assumptions relating to possible variations between the singlet and triplet radial wave functions. One involves the off-diagonal matrices of the spin-orbit energy that contributes to the effective values of the μ_2 parameters within each configuration. The second occurs in the transition matrix between configurations $\mathcal{M}_{\ell\ell'}$ and can affect the degree to which this quantity cancels for ratios of triplet-triplet and singlet-triplet transitions. The assumption of constancy can be tested through applications to measured lifetime data. Thus it is possible that even if $\mathcal{M}_{\ell\ell'}$ is not precisely the same for triplet-triplet and singlet-triplet transitions, the branching fractions may still be accurately specified by the formalism.

3. Results

Spectroscopic energy level data for the *nsn*p and *nsn*d levels in Ga II [11] ($n=4$), In II [12] ($n=5$) and Tl II [13] ($n=6$) were reduced to Slater parameters and mixing angles using Eqs. (3–9). These quantities are summarized in Table I. Notice that in the case of Tl II the effective values of both G_1(sd) and θ_2 are negative. This is the result of a homologous avoided crossing of the *nsn*d and *n*p^2 ^{1}D$_2$ levels that occurs between $n=5$ and 6. As discussed in Ref. [14], when energy levels undergo an avoided crossing in an isoelectronic or homologous sequence, their eigenvectors reverse roles, and should therefore be treated as having undergone an actual crossing for semiempirical formulations. Although this occurence is an indication of significant CI, the *n*p^2 ^{1}D$_2$ level lies well below all of the *nsn*d levels for $n=4$ and 5, and well above all the *nsn*d levels for $n=6$, and in such cases the CI is often manifested only indirectly in the effective value of mixing angle.

Table I. *Fitted parameters obtained from energy level data.*

Parameter	Ga II	In II	Tl II
G_1(sp)	11999.00	9140.41	8438.50
μ_1(sp)	921.49	2368.00	8182.67
μ_2(sp)	806.07	2075.20	7130.30
θ_1	1.427°	4.287°	12.842°
G_1(sd)	6165.28	5831.42	–691.50
μ_1(sd)	26.91	87.84	271.60
μ_2(sd)	72.46	119.84	97.80
θ_2	0.412°	0.718°	–5.437°

These mixing angle data were then used together with Eqs. (16–30) to compute the relative multiplet fractions R_{ik} and the branching fractions BF for the n^2-*nsn*p and *nsn*p-*nsn*d transitions, which are given in Table II (expressed as percentages to avoid leading zeros). For the case of In II, these transition probabilities and branching fractions are compared with the relativistic quantum defect orbital (RQDO) calculations of Lavín and Martin [15], and show excellent agreement. See Table III.

These results in Table II were also compared with the available data for lifetime measurements of the *nsn*p and *nsn*d levels in the Ga II, In II and Tl II ions as summarized in Table IV. With the exclusion of two obviously erroneous older measurements, the lifetime data are given in Table IV. Weighted averages were taken where multiple measurements exist and yielded values for $\tau(^1\mathrm{P}_1)$ of 0.46±0.02 ns in Ga II and 0.82±0.04 ns for In II, and a value for $\tau(^1\mathrm{D}_2)$ of 6.0±1.4 ns for Tl II.

For the sp lifetimes it can be seen that the triplet lifetimes can be accurately predicted from the singlet lifetimes using the relationship $\tau_{^3\mathrm{P}_1} = \tau_{^1\mathrm{P}_1} \times C_{^1\mathrm{P}_1}/C_{^3\mathrm{P}_1}$. As shown in Table IV, the comparison of predicted vs experimental triplet lifetimes yields 449 vs 440±40 ns for In II, and 34.2 vs 39±3 for Tl II. For comparison, recent calculations of the $^3\mathrm{P}_1$ lifetimes are given. Many theoretical calculations exist for the $^1\mathrm{P}_1$ lifetime, most of which agree reasonably well with measurements, so only the value for Tl II (a recent calculation reporting lifetimes for both the $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ levels) is given. These results give credence to the predicted value of 2380 ns for Ga II.

For the sd levels, the triplets can also be computed from the singlets using $\tau_{^3\mathrm{D}_2} = \tau_{^1\mathrm{D}_2} \times C_{^1\mathrm{D}_2}/C_{^3\mathrm{D}_2}$. This yields predictions somewhat smaller than the experimental values: 0.41 vs 0.67 ± 0.06 ns for Ga II; and 0.52 vs 0.91±0.03 ns for In II. Within the triplets predictions were made using $\tau_{^3\mathrm{D}_J} = \tau_{^1\mathrm{D}_2} \times C_{^1\mathrm{D}_2}/C_{^3\mathrm{D}_J}$. These yield good agreement between predicted and experiental values: 0.86 vs 0.86 ± 0.03 ns for $J=1$; and 1.00 vs 0.94 ± 0.03 ns for $J=3$.

The differences between predicted and measured singlet-from-triplet lifetime values are probably a result of CI. While the sp $^3\mathrm{P}_1$ and $^1\mathrm{P}_1$ levels and all of the sd $^3\mathrm{D}_J$ levels are virtually free of CI, the sd $^1\mathrm{D}_2$ level is subject to substantial CI from the p^2 $^1\mathrm{D}_2$ level. This could cause differences between the singlet-triplet and triplet-triplet values of $\mathcal{M}_{\ell\ell'}$ and thereby affect the prediction of the triplet lifetime made from that of the singlet. However, this may not affect the validity of the predictions of the triplet-to-triplet lifetimes and branching fractions.

On the basis of these considerations, Table IV uses the values of C_i in Table II to predict the triplet lifetimes of

Table II. *Wavelengths (in air for $\lambda \geq 2000$Å), multiplet fractions (in %), branching fractions (in %) and transition probability rates (in ns^{-1}) for the two supermultiplets. The transition probability rate predictions are based on the branching fractions obtained by this formalism and the measured and predicted lifetimes given in Table III.*

	Ga II				In II				Tl II			
Transition	λ(Å)	R_{ik}	BF	A_{ik}	λ(Å)	R_{ik}	BF	A_{ik}	λ(Å)	R_{ik}	BF	A_{ik}
$^1S_0$–$^3P_1$	2090.77	0.019	100	0.00042	2306.15	0.18	100	0.0023	1908.65	1.70	100	0.026
$^1S_0$–$^1P_1$	1414.40	99.98	100	2.19	1586.45	99.82	100	1.22	1321.70	98.30	100	1.69
$^3P_0$–$^3D_1$	1504.93	15.94	56.13	0.854	1672.00	16.57	57.17	0.665	1499.34	4.14	60.31	–
$^3P_1$–	1515.11	11.71	41.23	0.627	1702.57	11.70	40.38	0.470	1568.53	8.81	37.55	–
$^3P_2$–	1536.90	0.75	2.63	0.040	1777.57	0.69	2.38	0.028	1837.49	0.38	1.64	–
$^1P_1$–	2318.68	0.002	0.01	0.0001	2560.06	0.02	0.07	0.001	2469.18	0.12	0.50	–
SUM		28.40	100	1.522		28.98	100	1.163		23.45	100	–
$^3P_1$-$^3D_2$	1514.51	21.11	75.79	1.131	1700.08	21.20	77.22	0.849	1561.60	25.22	79.30	–
$^3P_2$–	1536.28	6.74	24.21	0.361	1774.86	6.23	22.70	0.249	1827.99	5.80	18.24	–
$^1P_1$–	2317.27	0.002	0.01	$<10^{-4}$	2554.44	0.023	0.08	0.001	2452.04	0.78	2.46	–
SUM		27.85	100	1.493		27.46	100	1.099		31.80	100	–
$^3P_2$-$^3D_3$	1535.31	27.03	100	1.448	1770.66	25.11	100	1.063	1814.85	33.49	100	–
$^3P_1$–$^1D_2$	1275.94	0.16	0.02	0.0003	1417.81	0.20	1.08	0.014	1593.19	3.21	28.52	0.048
$^3P_2$–	1291.36	$<10^{-3}$	$<10^{-3}$	10^{-5}	1469.44	0.002	0.01	0.0001	1871.43	0.05	0.44	0.001
$^1P_1$–	1802.25	16.70	99.98	1.460	1966.71	18.25	98.91	1.285	2530.88	7.99	71.04	0.118
SUM		16.72	100	1.460		18.45	100	1.299		11.25	100	0.167

Table III. *Comparison of predicted transition probabilities and branching fractions for the 5s5p ^3P–5s5d ^3D manifold in In II.*

	$A_{ik}(ns^{-1})$			BF(%)		
Transition	RQDO[a]	RQDO[b]	SE[c]	RQDO[a]	RQDO[b]	SE[c]
5s5p $^3P_0$–5s5d $^3D_1$	0.696	0.593	0.665	57.9	57.9	57.17
5s5p $^3P_1$–	0.483	0.412	0.470	40.2	40.2	40.38
5s5p $^3P_2$–	0.023	0.020	0.028	1.9	1.9	2.38
5s5p $^1P_1$–	–	–	0.001	–	–	0.07
5s5p $^3P_1$–5s5d $^3D_2$	0.873	0.747	0.849	80.1	80.1	77.22
5s5p $^3P_2$–	0.218	0.186	0.249	19.9	19.9	22.70
5s5p $^1P_1$–	–	–	0.001	–	–	0.08
5s5p $^3P_2$–5s5d $^3D_3$	0.878	0.754	1.063	100.	100.	100.

[a] Lavín and Martin, rel. quantum defect orbital [15].
[b] Lavín and Martin, rel. quantum defect orbital with polarization [15].
[c] This work.

Table IV. *Data base of experimental lifetime measurements (Exp) and semiempirical predictions (SE) based on this parametrization. Quoted measurement uncertainties are given in parentheses. The $^3P_1$ predictions are based on the $^1P_1$ measurements, and the $^3D_1$ and $^3D_3$ predictions are based on the $^3D_2$ measurements. A few recent calculations for the lifetimes of the nsnp levels (Theo) are included for comparison.*

	Ga II			In II			Tl II		
Level	Exp	Theo	SE[a]	Exp	Theo	SE[a]	Exp	Theo	SE[a]
$^3P_1$	–	2445[b]	2380	440(40)[c]	598[d]	449	39(3)[e]	36.3[f]	34.2
$^1P_1$	0.41(3)[g], 0.49(4)[h], 0.65(8)[i], 0.48(12)[j]		–	0.79(5)[k], 0.90(8)[l]		–	0.59(4)[e]	0.574[f]	–
$^3D_1$	–		0.66	0.86(3)[k]		0.86	–		–
$^3D_2$	0.67(6)[j]		–	0.91(3)[k]		–	–		–
$^3D_3$	–		0.69	0.94(3)[k]		1.00	–		–
$^1D_2$	0.67(4)[j], 0.73(7)[m]		–	0.77(3)[k]		–	5(1)[n], 7(1)[o]		–

[a] This work.
[b] Fleming and Hibbert [16].
[c] Peik *et al.* [19].
[d] Chou *et al.* [17].
[e] Henderson and Curtis [20].
[f] Brage et al [18].
[g] Engström [21].
[h] Andersen *et al.* [22].
[i] Sørensen [23].
[j] Ansbacher *et al.* [24].
[k] Ansbacher *et al.* [25].
[l] Andersen *et al.* [26].
[m] Denne *et al.* [27].
[n] Andersen and Sørensen [28].
[o] Shimon and Erdevdi [29].

the $J = 1,3$ levels from the $J = 2$ for Ga II, and combines these three lifetimes with the branching fractions to obtain the corresponding transition probabilities. For In II all of the sd lifetimes have been measured, and were used together with the branching fractions to obtain the transition probabilities tabulated. In the case of Tl II the measured lifetime of the sd singlet level was combined with the branching fractions to yield the transition probabilities tabulated. For the Tl II triplet levels, transition probabilities can be computed from the tabulated branching fractions when lifetime measurements become available.

In conclusion, this formalism provides a predictive systematization of lifetime and energy level data that is simple to use, contains internal checks of its validity, and permits predictions of lifetimes, branching fractions, and transition probabilities.

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