

# Isoelectronic wavelength predictions for magnetic-dipole, electric-quadrupole, and intercombination transitions in the Mg sequence

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Precise wavelength predictions are presented for the  $3s3p\ ^3P_1$ - $3s3p\ ^3P_2$  magnetic-dipole transitions, the  $3s3p\ ^3P_0$ - $3s3p\ ^3P_2$  electric-quadrupole transitions, and the  $3s^2\ ^1S_0$ - $3s3p\ ^3P_1$  intercombination lines for all members of the Mg isoelectronic sequence with  $Z \leq 45$ . Magnetic-dipole and intercombination transitions of this type have recently been observed in tokamak spectra and have applications for localized diagnostics in high-temperature plasmas. The predictions are based on interpolations and extrapolations of empirical regularities, which also provide insights into the dynamics of these atomic systems.

## INTRODUCTION

Radiative transitions between low-lying metastable states in highly ionized atoms have recently come under scrutiny because of their applications in tokamak plasma diagnostics.<sup>1</sup> States of the same principal quantum number as the ground state that are forbidden to decay at low  $Z$  by parity and spin-selection rules can become strongly emissive in highly ionized atoms. Such states are often heavily populated by growing-in cascades and have transition wavelengths in a spectral region where normal-incidence spectrometers with multiply reflective optics can be used.

A number of experimental and theoretical studies<sup>1-5</sup> have already been made of the magnetic-dipole transitions within the  $^2P$  ground-state terms of the B, F, Al, and Cl isoelectronic sequences, and measurements have recently been made for the lowest-lying  $^3P$  terms in the Mg sequence.<sup>5-7</sup> In pure  $LS$  coupling, the  $3s3p\ ^3P$  term could not decay by emission of  $E1$  radiation. However, relativistic spin mixing causes the intercombination line  $3s^2\ ^1S_0$ - $3s3p\ ^3P_1$  to become  $E1$  allowed, and the rate grows rapidly with increasing  $Z$ . Within the  $3s3p\ ^3P$  term, the magnetic-dipole transition  $^3P_1$ - $^3P_2$  can become strong at high  $Z$ , although the  $^3P_0$ - $^3P_1$  transition is weakened by competitive branching to ground via the intercombination line. At sufficiently high  $Z$  the electric-quadrupole transition  $^3P_0$ - $^3P_2$  might become competitive. The decay to ground of the  $^3P_0$  level remains forbidden to single-photon emission irrespective of  $Z$ .

Earlier<sup>8</sup> we developed a procedure for parameterizing existing data to obtain predictions for the energy levels of the  $3s3p\ ^1,^3P$  states in the Mg isoelectronic sequence that utilizes screening parameterizations of empirical spin-orbit and Slater energies. Since screening of the  $3p$  orbital alone dominates the isoelectronic variation of the electron-nucleus and spin-orbit energies, it was found that these quantities could be described well by simple screened one-electron formulas. However, for the electron-electron exchange integral there is no corresponding classical counterpart, and uncertainties in the extrapolation of this quantity were the limiting factor

in the extension of this formalism to high degrees of ionization.

A number of measurements for highly ionized members of this sequence were recently made. Wavelength measurements for the  $3s^2\ ^1S_0$ - $3s3p\ ^3P_1$  intercombination lines in high-charge states have been made<sup>6,7</sup> that reveal earlier misclassifications that shift the triplet states and ionization potentials in several ions by more than a kilokayser relative to the singlet states. Measurements of the singlet resonance transition have also been made<sup>9</sup> for very high stages of ionization, providing a nearly complete isoelectronic specification of excitation energies for the  $3s3p\ ^1P_1$  level up to  $Z = 45$ . This enables us to couple the  $3s3p$  configuration directly to the ground state without introducing the uncertainties in the ionization limit, as was necessary in Ref. 8. Measurements of the  $3s3p\ ^3P_1$ - $3s3p\ ^3P_2$  fine-structure separations have been made through the observation of magnetic-dipole transitions in highly charged ions.<sup>5</sup>

These new measurements<sup>5-7,9</sup> greatly improve the data base and have permitted us to test semiempirical models sensitively for parameterization of the exchange energy. A parameterized heliumlike model proposed by Edlén<sup>10</sup> has been found to describe the isoelectronic behavior of this quantity and permits accurate extrapolative predictions of the excitation energies and transition wavelengths of the  $3s3p$  configuration up to  $Z = 45$ .

## REDUCTION OF MEASURED DATA TO SPIN-ORBIT AND EXCHANGE ENERGIES

If we denote the excitation energies (in inverse centimeters) of the  $3s3p\ ^1,^3P$  levels above the  $3s^2\ ^1S_0$  ground state by their spectroscopic symbols  $^1,^3P_J$ , then the energy separations for the resonance line  $\Delta E_R$ , the intercombination line  $\Delta E_I$ , the magnetic-dipole transition  $\Delta E_{M1}$ , the electric-quadrupole transition  $\Delta E_{E2}$ , and the corresponding wavelengths  $\lambda_R$ ,  $\lambda_I$ ,  $\lambda_{M1}$ , and  $\lambda_{E2}$  are given by

$$\Delta E_R = 1/\lambda_R = ^1P_1, \quad (1)$$

$$\Delta E_I = 1/\lambda_I = {}^3P_1, \tag{2}$$

$$\Delta E_{E_2} = 1/\lambda_{E_2} = {}^3P_2 - {}^3P_0, \tag{3}$$

$$\Delta E_{M_1} = 1/\lambda_{M_1} = {}^3P_2 - {}^3P_1. \tag{4}$$

The energy separations defined by Eqs. (1)–(4) provide a better specification of the data base than do the excitation energies. This is because the triplet fine structures are determined from intratriplet transitions and are defined relative to the ground state only for cases for which the intercombination line has been measured. By using the intermediate coupling equations for a pure *nsnp* configuration (Ref. 11, p. 266), the triplet energy levels can be described in terms of the singlet energy level and two physically interpretable quantities: the exchange electrostatic electron–electron Slater energy  $G_1$  and the spin–orbit energy  $\zeta$ . Thus

$${}^3P_0 = {}^1P_1 - G_1 - 3\zeta/4 - Q, \tag{5}$$

$${}^3P_1 = {}^1P_1 - 2Q, \tag{6}$$

$${}^3P_2 = {}^1P_1 - G_1 + 3\zeta/4 - Q, \tag{7}$$

where

$$Q \equiv [(G_1 + \zeta/4)^2 + \zeta^2/2]^{1/2}. \tag{8}$$

The quantities  $G_1$  and  $\zeta$  are overdetermined and can be deduced from the energy separations of Eqs. (1)–(4) by using Eqs. (5)–(8) in a number of ways. One convenient procedure is to deduce the spin–orbit energy from

$$\zeta = 2\Delta E_{E_2}/3 \tag{9}$$

and the exchange energy from

$$2G_1 = \Delta E_R - \Delta E_I + \Delta E_{E_2} - 2\Delta E_{M_1}. \tag{10}$$

If, as in Ref. 8, the singlet energy  ${}^1P_1$  were also being parameterized, it would be necessary to include the electron–nucleus Balmer energy  $E_B$ , the direct electron–electron Slater energy  $F_0$ , and the ionization potential of the ground state  $IP$ . Since it is not possible to measure  $E_B$  and  $F_0$  separately, this both complicates the modeling and introduces large uncertainties that are due to  $IP$ . The  $3s3p$   ${}^1P_1$  excitation energies are now known nearly completely up to  $Z = 45$ , so we instead use here the singlet and the semiempirically parameterized  $\zeta$  and  $G_1$  values to predict the triplet energy levels.

### MODEL PARAMETERIZATIONS

It has long been established that the spin–orbit energy of the Mg isoelectronic sequence can be effectively parameterized through the regular doublet law.<sup>12</sup> This consists of mapping the observed spin–orbit intervals into a parameter  $S_\zeta$  using a screened Sommerfeld formula,<sup>10,13,14</sup> which is, for a  $3p$  level,

$$\zeta = \frac{R\alpha^2}{81(1 + \epsilon_\zeta)} [(Z - S_\zeta)^4 + \text{higher order}]. \tag{11}$$

Here  $R$  is the reduced-mass-corrected Rydberg constant,  $\alpha$  is the fine-structure constant,  $Z$  is the nuclear charge, and  $S_\zeta$  is an empirical screening parameter that this equation serves to define. A number of attempts have recently been made to improve the model and to widen its applicability. The parameter  $\epsilon_\zeta$  has been introduced<sup>3,4</sup> and can be adjusted to maximize the isoelectronic regularity of  $S_\zeta$  for certain se-

quences. However, for the Mg sequence considered here, the data are best fitted by  $\epsilon_\zeta = 0$ . This procedure has also been extended<sup>13,14</sup> through the inclusion of higher-order terms in the Dirac energy, corrections for the anomalous magnetic moment of the electron, and other radiative corrections. Details are given in Refs. 3, 4, 13, and 14. Here, as in other systems studied earlier,<sup>3,4,10,14</sup> this mapping of data yields values for  $S_\zeta$  that are accurately described by the empirical relationship

$$S_\zeta = a_\zeta + b_\zeta/(Z - S_\zeta). \tag{12}$$

A plot of  $S_\zeta$  versus  $1/(Z - S_\zeta)$  is given in Fig. 1 of Ref. 8 and clearly demonstrates the empirical regularity of this parameterization.

The semiempirical description of the electrostatic electron–electron–exchange interaction is complicated by the fact that there is no simple classical or one-electron model for this quantity since the screening of both the  $s$  and the  $p$  electrons would be expected to be important here and since exchange is not a classical operation. A calculation of the Slater integrals (Ref. 11, p. 197) using hydrogenic wave functions and an effective central charge  $Z - S$  (assuming that both the  $ns$  and the  $np$  orbitals with either  $1/r >$  or  $r < /r >^2$  weighting feel the same effective central charge) yields

$$F_0(nsnp) = f(nsnp)(Z - S) \tag{13}$$

and

$$G_1(nsnp) = g(nsnp)(Z - S) \tag{14}$$

where  $f(nsnp)$  and  $g(nsnp)$  are rational fractions whose values have been calculated by us and are presented in Table 1. The quantities were computed numerically, and the rational fraction equivalents were assigned to within seven-figure accuracies. Although only the  $n = 3$  exchange value will be used here, the tabulation includes direct and exchange values for  $n = 2$ –6 and could be used to extend this procedure to the Be, Mg, Zn, Cd, and Hg isoelectronic sequences.

Edlén<sup>10</sup> has used Eq. (14) to convert empirical  $G_1$  values to corresponding  $S$  values, which he found exhibited a roughly linear behavior as a function of an appropriately defined reciprocal screened charge. He theoretically computed the coefficient  $g(2s2p) = 15R/256$  for the Be sequence but used  $f(nsnp)$  as a fitting parameter for the Mg, Zn, Cd, and Hg sequences. Following this procedure, we have reduced the empirical values of  $G_1$  to values for  $S_G$  by using

$$G_1 = \frac{g(3s3p)}{(1 + \epsilon_G)} (Z - S_G) \tag{15}$$

and allowed for empirical corrections to theoretical value  $g(3s3p) = 65R/2304$  through a parameter  $\epsilon_G$ , adjusted to maximize the isoelectronic regularity of  $S_G$ . Here again it was

**Table 1. Direct and Exchange Slater Integrals for Screened Hydrogenic Wave Functions (in Rydberg units)**

$n$	$f(nsnp)$ × Denominator	$g(nsnp)$ × Denominator	Denominator
2	83	15	256
3	317	65	2 304
4	6 573	1 409	86 400
5	3 651	799	75 600
6	43 269	9 577	1 296 000

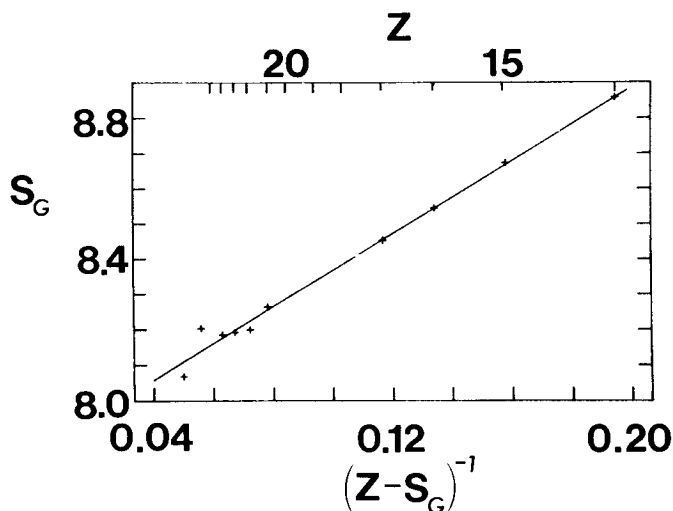


Fig. 1. Plot of the exchange energy screening parameter versus the reciprocal screened charge for  $\epsilon_G = 0.065$ .

found that the data are well represented by the relationship

$$S_G = a_G + b_G/(Z - S_G). \tag{16}$$

Figure 1 displays experimental data reduced in this manner, and here again the linearity of the exchange screening parameter as a function of reciprocal screened charge is evident.

**DATA SOURCES**

The primary data on which this analysis is based are presented, with source references,<sup>5-7,9,12,15-35</sup> in Table 2. The data are expressed as the energy separations  $\Delta E_R$ ,  $\Delta E_I$ ,  $\Delta E_{E2}$ , and  $\Delta E_{M1}$  because these quantities correspond more closely to the actual measurements than do the excitation or term energies. Comprehensive spectroscopic analyses that include intersystem measurements are available only for the ions from Mg I through S VI.<sup>15-19</sup> Measurements of the resonance transition are available for all ions with  $Z \leq 45$  except Br, Rb, and Tc, whereas measurements of the intercombination line

**Table 2. Experimental Data Base (in  $\text{cm}^{-1}$ ) with Source References**

Z	Ion	$^1P_1$	$^3P_1$	$^3P_2-^3P_0$	$^3P_2-^3P_1$
12	Mg I	35 051.264 (Ri) <sup>a</sup>	21 870.464 (Ri) <sup>a</sup>	60.773 (Ri) <sup>a</sup>	40.714 (Ri) <sup>a</sup>
13	Al II	59 852.02 (KH) <sup>a</sup>	37 453.91 (KH) <sup>a</sup>	184.76 (KH) <sup>a</sup>	123.88 (KH) <sup>a</sup>
14	Si III	82 884.41 (To)	52 853.28 (To)	390.32 (To)	261.73 (To)
15	P IV	105 190.42 (ZM)	68 146.48 (ZM)	697.14 (ZM)	468.69 (ZM)
16	S V	127 150.7 (JZ)	83 393.5 (JZ)	1 131.2 (JZ)	761.7 (JZ)
17	Cl VI	148 949 (BM)	98 632.1 (Ju)	1 718 (PP1)	1 165 (PP1)
18	Ar VII	170 720 (PP2)		2 486 (PP2)	1 681 (PP2)
19	K VIII	192 537 (Ek)		3 485 (Ek)	2 373 (Ek)
20	Ca IX	214 482 (Ek)		4 736 (Ek)	3 237 (Ek)
21	Sc X	236 490 (Ek)	160 128 (Fi2)	6 313 (Ek)	4 307 (Ek)
22	Ti XI	258 973 (Ek)	175 654 (Fi1)	8 199 (Ek)	5 633 (Ek)
23	V XII	281 603 (FP)	191 424 (Fi1)	10 250 (SC1)	7 160 (SC1)
24	Cr XIII	304 609 (SC2)	207 383 (Fi1)	13 120 (SC2)	9 190 (SC2)
25	Mn XIV	328 030 (Fa)		16 180 (CS)	11 400 (CS)
26	Fe XV	351 914 (BC)	239 660 (BC)	20 040 (FK)	14 400 (FK)
27	Co XVI	376 310 (SC3)		24 390 (SC3)	17 400 (SC3)
28	Ni XVII	401 320 (FH1)	272 702 (Fi2)	29 210 (FK)	20 960 (FK)
29	Cu XVIII	426 913 (FH2)	289 352 (Fi2)	36 120 (FK)	25 363 (De)
30	Zn XIX	453 309 (FH2)		52 025 (KJ) <sup>a</sup>	37 555 (KJ) <sup>a</sup>
31	Ga XX	480 515 (FH2)			
32	Ge XXI	508 725 (FH2)	340 832 (Fi2)		42 537 (De)
33	As XXII	538 010 (FH2)			
34	Se XXIII	568 440 (FH2)	376 364 (Fi2)		58 340 (De)
35	Br XXIV				
36	Kr XXV	629 000 (Hi)			
37	Rb XXVI				
38	Sr XXVII	702 060 (Re)			
39	Y XXVIII	739 560 (Re)			
40	Zr XXIX	778 340 (Re)	486 855 (Fi2)		134 862 (De)
41	Nb XXX	819 180 (Re)			
42	Mo XXXI	862 140 (Re)	524 934 (Fi2)		173 160 (De)
43	Tc XXXII				
44	Ru XXXIII	954 610 (Re)			
45	Rh XXXIV	1 04 350 (Re)			

<sup>a</sup> Not used in fitting. Key to references: (BC), Behring *et al.*, Ref. 29. (BM), Bowen and Millikan, Ref. 12. (CS), Corliss and Sugar, Ref. 28. (De), Denne *et al.*, Ref. 5. (Ek), Ekberg, Ref. 23. (Fa), Fawcett, Ref. 27. (FH1), Fawcett and Hayes, Ref. 32. (FH2), Fawcett and Hayes, Ref. 33. (FK), Feldman *et al.*, Ref. 30. (Fi1), Finkenthal *et al.*, Ref. 6. (Fi2), Finkenthal *et al.*, Ref. 7. (FP), Fawcett and Peacock, Ref. 24. (Hi), Hinnov, Ref. 35. (JZ), Joelsson *et al.*, Ref. 19. (Ju), Jupén, Ref. 20. (KH), Kaufman and Hagan, Ref. 16. (KJ), Khan *et al.*, Ref. 34. (PP1), Parker and Phillips, Ref. 21. (PP2), Phillips and Parker, Ref. 22. (Re), Reader, Ref. 9. (Ri), Risberg, Ref. 15. (SC1), Sugar and Corliss, Ref. 25. (SC2), Sugar and Corliss, Ref. 26. (SC3), Sugar and Corliss, Ref. 31. (To), Toresson, Ref. 17. (ZM), Zetterberg and Magnusson, Ref. 18.

**Table 3. Fitting Parameters**

Quantity	$i$	$\epsilon_i$	$a_i$	$b_i$
Fine Structure	$\zeta$	0.000	4.782	11.40
Exchange	$G$	0.065	7.849	5.18

are available for only half of these 34 ions. Thus most of the triplet data consist of fine-structure separations deduced from measurements of transitions from higher-lying  $^3S$  states made by one set of authors and intercombination lines from another set of authors. Because uncertainty estimates were not given by all the authors, and the source material is of a varied nature, uncertainties were assumed to be either constant or a fixed percentage of the measured quantity.

## RESULTS

The parameters  $\epsilon_i$ ,  $a_i$ , and  $b_i$  were obtained through the use of a nonlinear least-squares program<sup>36</sup> and yielded the values presented in Table 3. Elements at the neutral end of an isoelectronic sequence are usually more subject than others to perturbations from excited core configurations, so Mg I and Al II were excluded from the fit. The Zn XIX fine structures inferred from wavelength data in Ref. 34 were also excluded from the fit since they were more than  $10\,000\text{ cm}^{-1}$  off the trend and presumably are in error.

Our fit for  $\zeta$  is essentially equivalent to that of Ref. 8, although it utilizes a few additional data points. The linearity of  $S_\zeta$  as a function of  $1/(Z - S_\zeta)$  shown in Fig. 1 of Ref. 8 is retained through 18 stages of ionization, and we have extrapolated this trend through 34 stages. Two measurements deviate sufficiently from the interpolated trend to warrant restudy. We suggest that the values for  $\Delta E_{E2}$  in Table 2 be increased for V XI to  $10\,445\text{ cm}^{-1}$  and decreased for Cu XVIII to  $35\,113\text{ cm}^{-1}$ .

It should be noted that a similar screening parameter study<sup>14</sup> for the  $^2P$  terms in the Cu isoelectronic sequence indicated that the linear extrapolation began to underestimate slightly the measured fine-structure splittings<sup>37</sup> for members

of the sequence more than 30-times ionized. At 45-times ionized W this correction was still only about 1%, but it must be included for high-precision work. Additional measurements are needed to determine whether similar 1% corrections might be necessary for extrapolations beyond 30-times ionized members in the Mg sequence. Self-consistency checks between the spin-orbit and exchange parameterizations are described below that indicate that the extrapolations are accurate for the degrees of ionization considered here.

Figure 1 shows the linearity of  $S_G$  as a function of  $1/(Z - S_G)$  for the best-fitted value  $\epsilon_G = 0.065$ . The fit is good for all points except for Cu XVIII, which was excluded from the fit because, again here as in the spin-orbit parameterization, it was far off the trend. In view of the tendencies for a slight downward trend at high  $Z$  for fine-structure screening parameterizations of other sequences noted above, we attempted to make consistency checks for our  $Z > 28$  predictions.

If we assume that our extrapolations for  $S_\zeta$  are valid, then it is possible to deduce values for  $\zeta$  for Ge XXI, Se XXII, Zr XXIX, and Mo XXXI from the data base since only  $\Delta E_{E2}$  (the  $J = 2$  to  $J = 0$  fine-structure splitting) is lacking for full specification of  $3s3p$  in those ions. We did this and found that these four quasi-empirical data points do indicate a slight downward trend from the straight line in Fig. 1 for  $Z > 28$ . The corrected screening parameter  $S_G(\text{corr})$  could be represented by

$$S_G(\text{corr}) = S_G(\text{pred}) - 0.002(Z - 23)^2, \quad (Z > 28), \quad (17)$$

where  $S_G(\text{pred})$  is the value predicted by Eq. (16). This correction is small and alters the extrapolated wavelengths by less than one angstrom. The screening parameterization representation can sometimes reveal subtle dynamical effects too small to be apparent in the energy representation. Thus this correction may indicate the onset of a new regime of charge penetration as seen by either the exchange energy or the spin-orbit energy or both. Alternatively, it may be a result of uncertainties in the measurements or the simplicity of the model. The striking regularity exhibited in Fig. 1 provides additional motivation for further measurements in highly ionized members of this sequence.

**Table 4. Wavelengths of the Resonance, Intercombination, Magnetic-Dipole, and Electric-Quadrupole Transitions (in angstroms)**

$Z$	Ion	$R$ Obs.	$I$		$E2$		$M1$	
			Obs.	Pred.	Obs.	Pred.	Obs.	Pred.
29	Cu XVIII	234.24	345.6	345.8	2 848.0		3 941.6	3 932.7
30	Zn XIX	220.60		326.7	2 402.8			3 288.5
31	Ga XX	208.11		309.4	2 041.4			2 768.2
32	Ge XXI	196.57	293.4	293.6	1 745.4	2 350.2		2 344.6
33	As XXII	185.87		279.1	1 501.2			1 997.0
34	Se XXIII	175.92	265.7	265.8	1 298.1	1 714.1		1 710.0
35	Br XXIV				1 128.2			1 471.4
36	Kr XXV	159.0		244.7	985.1			1 272.0
37	Rb XXVI				863.8			1 104.4
38	Sr XXVII	142.438		222.8	760.6			962.9
39	Y XXVIII	135.216		213.8	672.2			842.7
40	Zr XXIX	128.479	205.4	205.6	596.3	741.5		740.3
41	Nb XXX	122.074		197.9	530.6			652.7
42	Mo XXXI	115.991	190.5	190.6	473.7	577.5		577.3
43	Tc XXXII				424.2			512.3
44	Ru XXXIII	104.755		177.3	380.8			456.1
45	Rh XXXIV	99.567		171.3	342.9			407.1

**Table 5. Observed and Predicted Energy Levels for the  $3s3p\ ^3P$  Term in the Mg Isoelectronic Sequence**

Z	Ion	$^3P_0$		$^3P_1$		$^3P_2$	
		Obs.	Pred.	Obs.	Pred.	Obs.	Pred.
12	Mg I	21 850.405		21 870.464		21 911.178	
13	Al II	37 393.02		37 453.91		37 577.79	
14	Si III	52 724.69	52 718	52 853.28	52 848	53 115.01	53 110
15	P IV	67 918.03	67 904	68 146.48	68 133	68 615.17	68 599
16	S V	83 024.0	83 041	83 393.5	83 410	84 155.2	84 168
17	Cl VI	98 073.7	98 111	98 632.1	98 670	99 794	99 825
18	Ar VII		113 147		113 952		115 635
19	K VIII		128 159		129 277		131 641
20	Ca IX		143 173		144 679		147 907
21	Sc X	158 122	158 081	160 128	160 060	164 435	164 363
22	Ti XI	173 101	173 249	175 654	175 794	181 300	181 420
23	V XII	188 334	188 303	191 424	191 515	198 584	198 748
24	Cr XIII	203 453	203 433	207 383	207 420	216 573	216 585
25	Mn XIV		218 620		223 498		234 965
26	Fe XV	233 910	233 866	239 660	239 754	253 820	253 944
27	Co XVI		249 161		256 185		273 570
28	Ni XVII	264 422	264 546	272 702	272 833	293 682	293 945
29	Cu XVIII		279 535	289 352	289 220		314 648
30	Zn XIX		294 916		306 126		336 534
31	Ga XX		310 351		323 213		359 337
32	Ge XXI		325 959	340 832	340 600		383 252
33	As XXII		341 726		358 266		408 340
34	Se XXIII		357 634	376 364	376 189		434 669
35	Br XXIV		373 577		394 255		462 216
36	Kr XXV		385 762		408 665		487 280
37	Rb XXVI		408 650		433 870		524 415
38	Sr XXVII		421 195		448 815		552 672
39	Y XXVIII		437 664		467 760		586 422
40	Zr XXIX		453 780	486 855	486 418		621 494
41	Nb XXX		470 204		505 442		658 660
42	Mo XXXI		486 874	524 934	524 761		697 973
43	Tc XXXII		506 439		547 017		742 204
44	Ru XXXIII		520 619		563 922		783 197
45	Rh XXXIV		537 653		583 710		829 323

In Table 4 we present the wavelength predictions for the intercombination lines, the  $J = 2$  to  $J = 0$  electric-quadrupole transition, and the  $J = 2$  to  $J = 1$  magnetic-dipole transition for  $Z = 29$ –45, along with the available direct measurements of these quantities and the observed wavelengths of the resonance transitions.

In Table 5 we present our prediction for the excitation energies of the  $3s3p$  levels relative to the ground state together with the available observations (source references are as in Table 2).

## CONCLUSIONS

Screened one-electron formulas have been utilized to map the spin-orbit and exchange energies of the  $3s3p$  configuration of the Mg sequence into a space in which they are isoelectronically linear. This permits precise interpolations and extrapolations of the excitation energies of the triplet states of this configuration relative to the singlet state. This also provides reliable estimates for the wavelengths for magnetic-dipole and electric-quadrupole transitions within the triplet term and for the intercombination transition to the ground state. The linearities observed here typify the dynamical properties of a large body of data and should be demanded of any *ab initio* theoretical model that is used to de-

scribe this sequence. The methods used could be further sharpened by measurement at higher  $Z$  and applied to other  $nsnp$  isoelectronic sequences.

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