Chapter 6

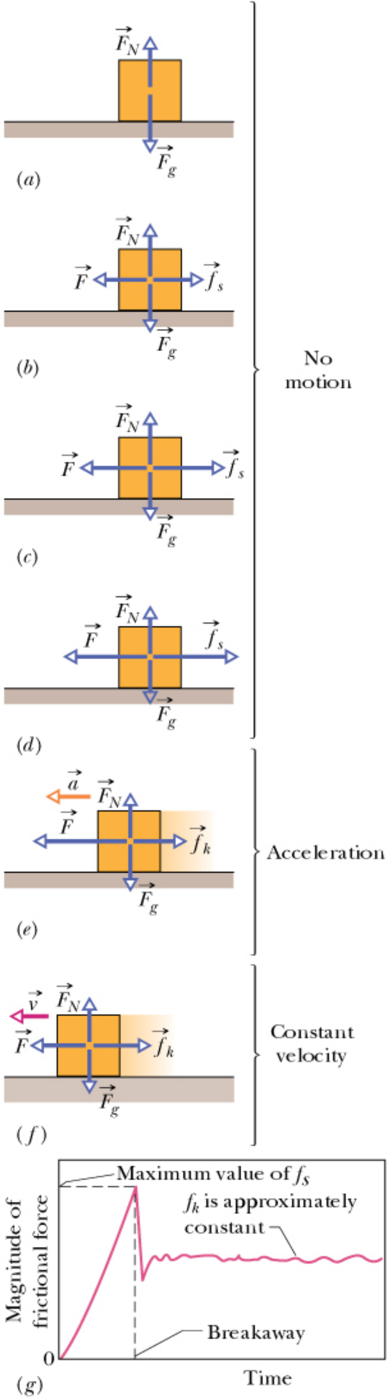
Force and Motion II

In this chapter we will cover the following topics:

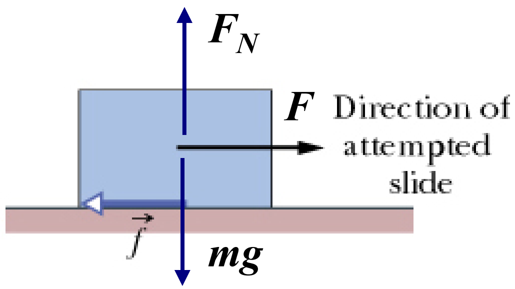
Describe the frictional force between two objects. Differentiate between static and kinetic friction, study the properties of friction, and introduce the coefficients for static and kinetic friction.

Study the drag force exerted by a fluid on an object moving through the fluid and calculate the terminal speed of the object.

Revisit uniform circular motion and using the concept of centripetal force apply Newton's second law to describe the motion.



Friction: We can explore the basic properties of friction by analyzing the following experiment based on our every day experience. We have a heavy crate resting on the floor. We push the crate to the left (frame b) but the crate does not move. We push harder (frame c) and harder (frame d) and the crate still does not move. Finally we push with all our strength and the crate moves (frame e). The free body diagrams for frames a-e show the existence of a new force $\vec{f}_s$ which balances the force $\vec{F}$ with which we push the crate. This force is called the static frictional force. As we increase F , f_s also increases and the crate remains at rest. When F reaches a certain limit the crate "breaks away" and accelerates to the left. Once the crate starts moving the force opposing its motion is called the kinetic frictional force $\vec{f}_k$. $f_k < f_s$. Thus if we wish the crate to move with constant speed we must decrease F so that it balances f_k (frame f). In frame (g) we plot f versus time t



$\boxed{f_{s,\max} = \mu_s F_N}$ $\boxed{0 < f_s \le \mu_s F_N}$ $\boxed{f_k = \mu_k F_N}$ (6-3)

Properties of friction: The frictional force is acting between two dry unlubricated surfaces in contact

Property 1. If the two surfaces do not move with respect to each other, then the static frictional force $\vec{f}_s$ balances the applied force $\vec{F}$.

Property 2. The magnitude f_s of the static friction is not constant but varies from 0 to a maximum value $f_{s,\max} = \mu_s F_N$ The constant μ_s is known as the coefficient of static friction. If F exceeds $f_{s,\max}$ the crate starts to slide

Property 3. Once the crate starts to move the frictional force $\vec{f}_k$ is known as kinetic friction. Its magnitude is constant and is given by the equation: $f_k = \mu_k F_N$
μ_k is known as the coefficient of kinetic friction. We note that: $f_k < f_{s,\max}$

Note 1: The static and kinetic friction acts parallel to the surfeces in contact
The direction opposes the direction of motion (for kinetic friction) or of attempted motion (in the case of static friction)

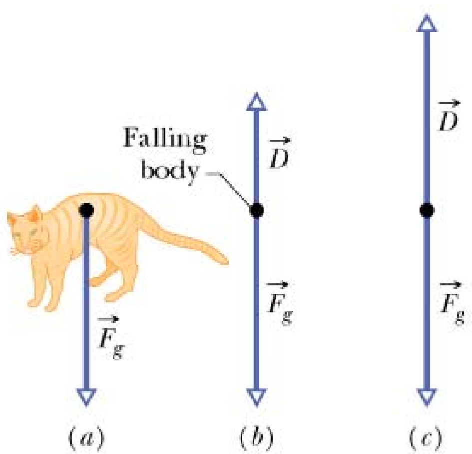
Note 2: The coefficient μ_k does not depend on the speed of the sliding object

Drag force and terminal Speed

When an object moves through a fluid (gas or liquid) it experiences an opposing force known as "drag". Under certain conditions (the moving object must be blunt and must move fast so as the flow of the liquid is turbulent) the magnitude of the drag force is given by the expression:

$$\boxed{D = \frac{1}{2} C \rho A v^2}$$

Here C is a constant , A is the effective cross sectional area of the moving object, ρ is the density of the surrounding fluid, and *v* is the object's speed. Consider an object (a cat of mass *m* in this case) start moving in air. Initially $D = 0$. As the cat accelerates D increases and at a certain speed v_t $D = mg$ At this point the net force and thus the acceleration become zero and the cat moves with constant speed v_t known the the **terminal speed**

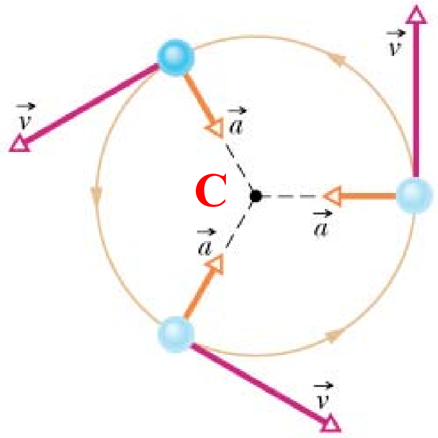


$$D = \frac{1}{2} C \rho A v_t^2 = mg$$

$$\boxed{v_t = \sqrt{\frac{2mg}{C \rho A}}}$$

(6-5)

Uniform Circular Motion, Centripetal force

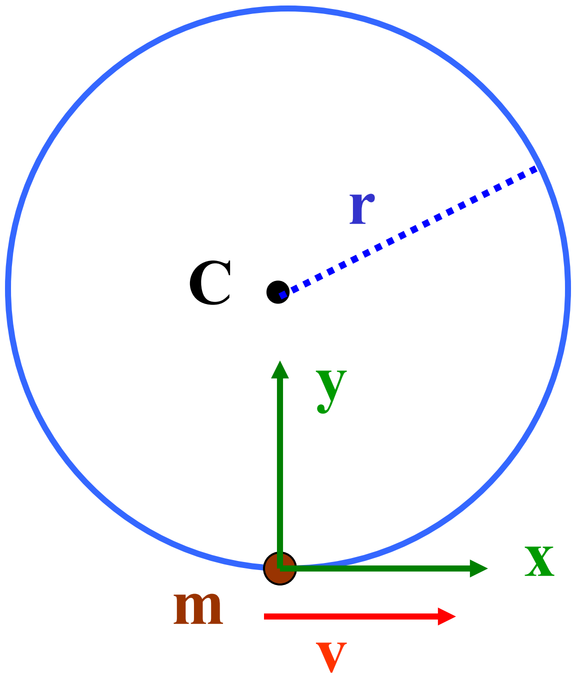


In chapter 4 we saw that an object that moves on a circular path of radius r with constant speed v has an acceleration a. The direction of the acceleration vector always points towards the center of rotation C (thus the name **centripetal**) Its magnitude is constant and is given by the equation: $\boxed{a = \dfrac{v^2}{r}}$

If we apply Newton's law to analyze uniform circular motion we conclude that the **net force in the direction that points towards C** must have

magnitude: $\boxed{F = \dfrac{mv^2}{r}}$ This force is known as "**centripetal force**"

The notion of centripetal force may be confusing sometimes. A common mistake is to "invent" this force out of thin air. Centripetal force is **not** a new kind of force. It is simply the **net force** that points from the rotating body to the rotation center C. Depending on the situation the centripetal force can be friction, the normal force or gravity. We will try to clarify this point by analyzing a number of examples

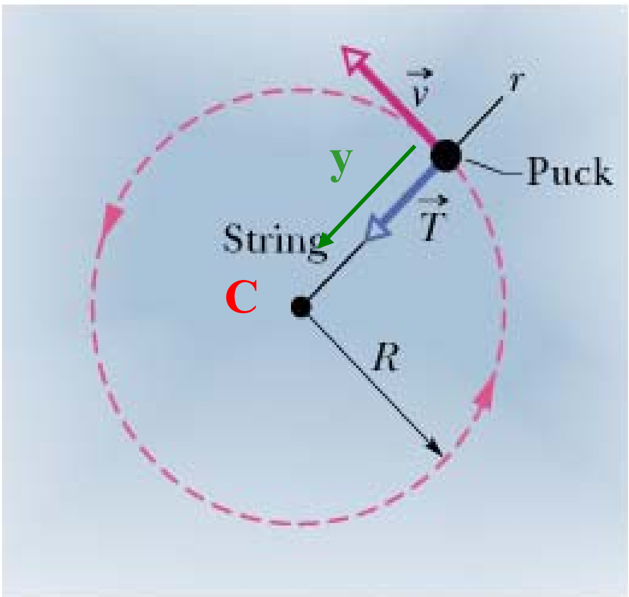


Recipe for problems that involve uniform circular motion of an object of mass m on a circular orbit of radius r with speed v

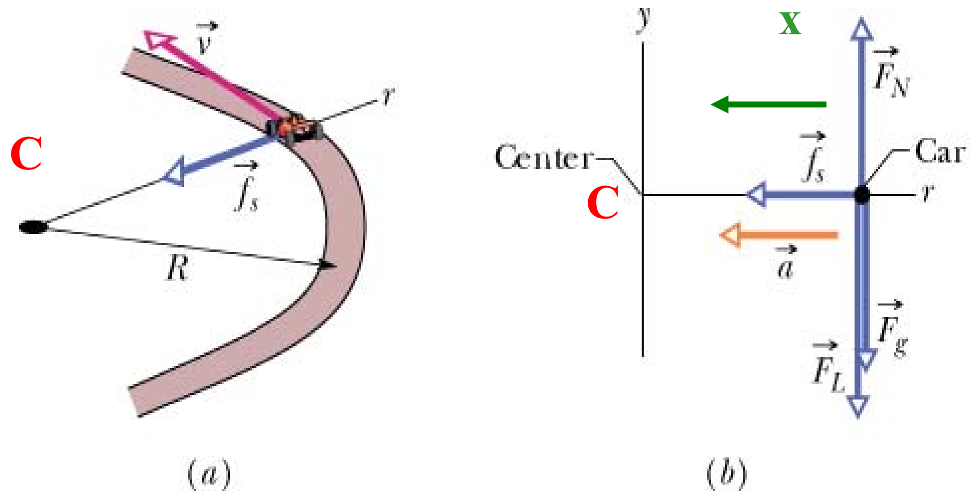
- Draw the force diagram for the object
- Choose one of the coordinate axes (the y-axis in this diagram) to point towards the orbit center C
- Determine F_{ynet}
- Set $F_{ynet} = \dfrac{mv^2}{r}$

A hockey puck moves around a circle at constant speed v on a horizontal ice surface. The puck is tied to a string looped around a peg at point C. In this case the net force along the y-axis is the tension T of the string. Tension T is the centripetal force.

Thus: $$F_{ynet} = T = \frac{mv^2}{r}$$



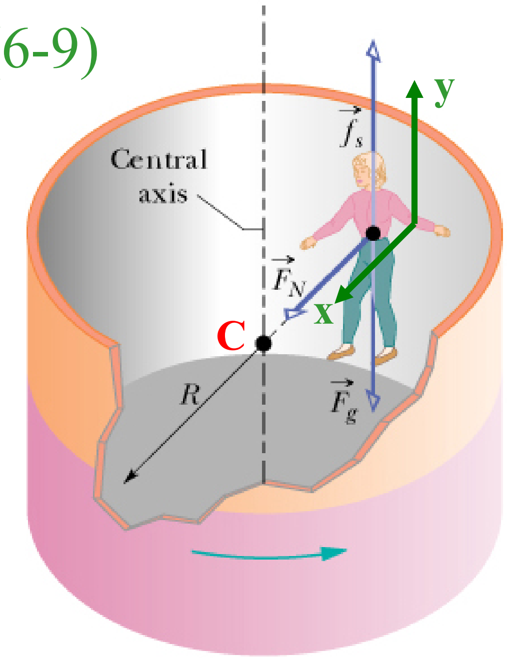
Sample problem 6-9: A race car of mass m travels on a flat circular race track of radius R with speed v. Because of the shape of the car the passing air exerts a downward force F_L on the car



If we draw the free body diagram for the car we see that the net force along the x-axis is the static friction f_s. The frictional force f_s is the centripetal force.

Thus: $$F_{xnet} = f_s = \frac{mv^2}{R} \qquad (6\text{-}8)$$

(6-9)



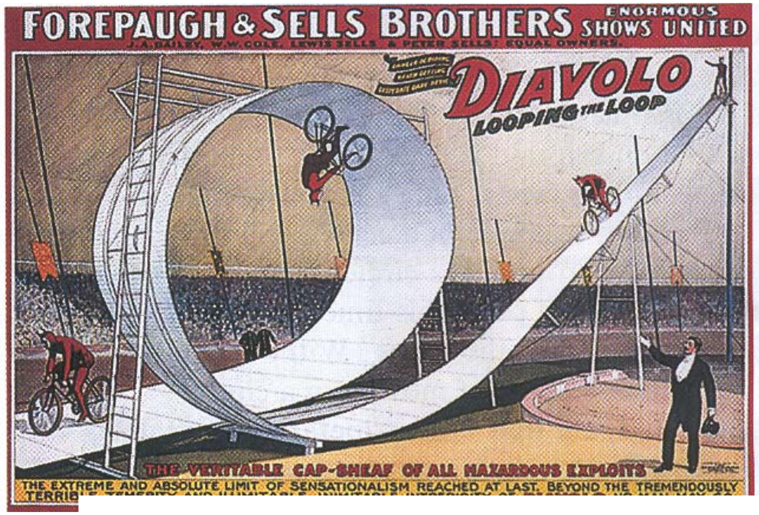
Sample problem 6-8: The Rotor is a large hollow cylinder of radius *R* that is rotated rapidly around its central axis with a speed *v*. A rider of mass *m* stands on the Rotor floor with his/her back against the Rotor wall. Cylinder and rider begin to turn. When the speed *v* reaches some predetermined value, the Rotor floor abruptly falls away. The rider does not fall but instead remains pinned against the Rotor wall. The coefficient of static friction μ_s between the Rotor wall and the rider is given.

We draw a free body diagram for the rider using the axes shown in the figure. The normal reaction F_N is the centripetal force.

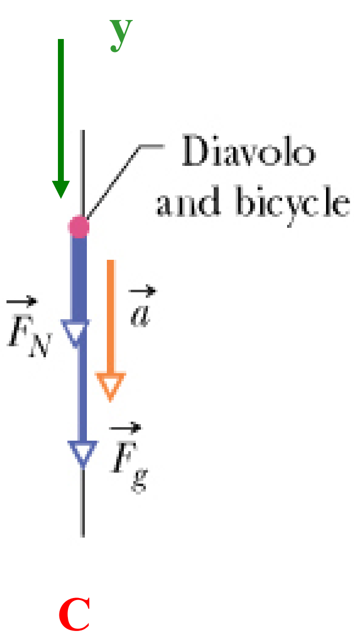
$$F_{x,net} = F_N = ma = \frac{mv^2}{R} \quad \text{(eqs.1)} \quad ,$$

$$F_{y,net} = f_s - mg = 0 \quad , \quad f_s = \mu_s F_N \rightarrow mg = \mu_s F_N \quad \text{(eqs.2)}$$

If we combine eqs.1 and eqs.2 we get: $mg = \mu_s \dfrac{mv^2}{R} \rightarrow v^2 = \dfrac{Rg}{\mu_s} \rightarrow v_{\min} = \sqrt{\dfrac{Rg}{\mu_s}}$



Sample problem 6-7: In a 1901 circus performance Allo Diavolo introduced the stunt of riding a bicycle in a looping-the-loop. The loop sis a circle of radius R. We are asked to calculate the minimum speed v that Diavolo should have at the top of the loop and not fall We draw a free body diagram for Diavolo when he is at the top of the loop. Two forces are acting along the y-axis: Gravitational force F_g and the normal reaction F_N from the loop. When Diavolo has the minimum speed v he has just lost contact with the loop and thus $F_N = 0$. The only force acting on Diavolo is F_g The gravitational force F_g is the centripetal force.



Thus: $$F_{ynet} = mg = \frac{mv_{\min}^2}{R} \rightarrow v_{\min} = \sqrt{Rg}$$