Chapter 5

Force and Motion

In chapters 2 and 4 we have studied "kinematics" i.e. described the motion of objects using parameters such as the position vector, velocity and acceleration without any insights as to what caused the motion. This is the task of chapters 5 and 6 in which the part of mechanics known as "dynamics" will be developed. In this chapter we will introduce Newton's three laws of motion which is at the heart of classical mechanics. We must note that Newton's laws describe physical phenomena of a vast range. For example Newton's laws explain the motion of stars and planets. We must also note that Newton's laws fail in the following two circumstances:

1. When the speed of objects approaches (1% or more) the speed of light in vacuum (c = 8×10^8 m/s). In this case we must use Einstein's special theory of relativity (1905)

2. When the objects under study become very small (e.g. electrons, atoms etc) In this case we must use quantum mechanics (1926)   (5-1)

Newton's First Law

Scientists before Newton thought that a force (the word "**influence**" was used) was required in order to keep an object moving at constant velocity. An object was though to be in its "natural state" when it was at rest. This mistake was made before **friction** was recognized to be a force. For example, if we slide an object on a floor with an initial speed v_o very soon the object will come to rest. If on the other hand we slide the same object on a very slippery surface such as ice, the object will travel a much larger distance before it stops. Newton checked his ideas on the motion of the moon and the planets. In space there is no friction, therefore he was able to determine the correct form of what is since known as : "**Newton's first law**"

**If the no force acts on a body, the body's velocity cannot change; that is the body cannot accelerate**

Note: If several forces act on a body (say $\vec{F}_A$, $\vec{F}_B$, and $\vec{F}_C$) the net force $\vec{F}_{net}$

is defined as: $\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$ i.e. $\vec{F}_{net}$ is the vector sum of $\vec{F}_A, \vec{F}_B,$ and $\vec{F}_C$

Force: The concept of force was tentatively defined as a push or pull exerted on an object. We can define a force exerted on an object quantitatively by measuring the acceleration it causes using the following procedure
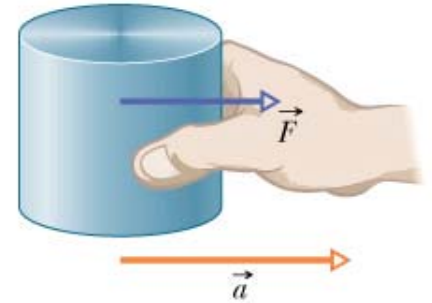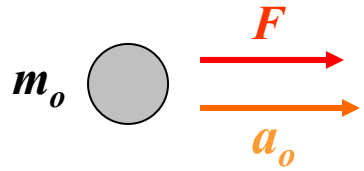
We place an object of mass $m = 1$ kg on a frictionless surface and measure the acceleration a that results from the application of a force F. The force is adjusted so that $a = 1$ m/s². We then say that $F = 1$ newton (symbol: N)

Note: If several forces act on a body (say $\vec{F}_A$, $\vec{F}_B$, and $\vec{F}_C$) the net force $\vec{F}_{net}$ is defined as: $\vec{F}_{net} = \vec{F}_A + \vec{F}_B + \vec{F}_C$ i.e. $\vec{F}_{net}$ is the vector sum of $\vec{F}_A, \vec{F}_B,$ and $\vec{F}_C$

(5-3)

m_o F a_o
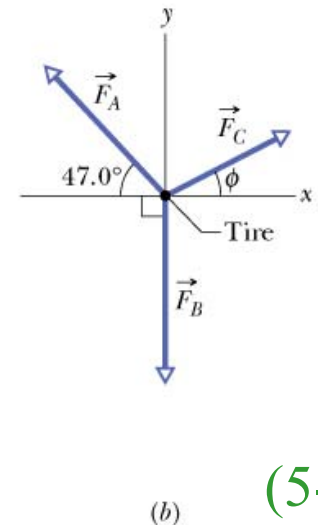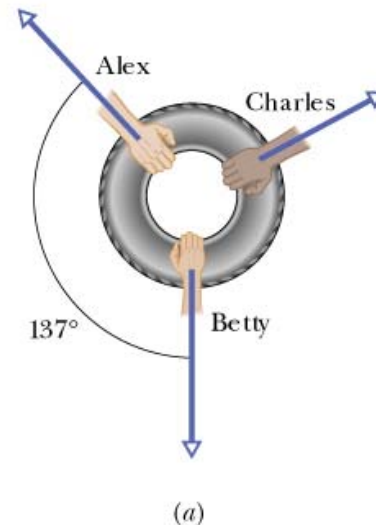
Mass: Mass is an ***intrinsic*** characteristic of a body that automatically comes with the existence of the body. But what is it exactly? It turns out that mass of a body is the characteristic that relates a force F applied on the body and the resulting ***acceleration a.***

m_X F a_X
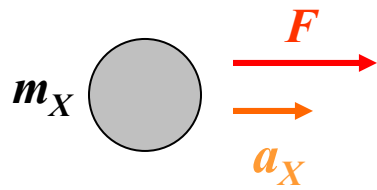
Consider that we have a body of mass $m_o = 1$ kg on which we apply a force $F = 1$ N. According to the definition of the newton , F causes an acceleration $a_o = 1$ m/s². We now apply F on a second body of unknown mass m_X which results in an acceleration a_X . The ratio of the accelerations is inversely proportional to the ratio of the masses
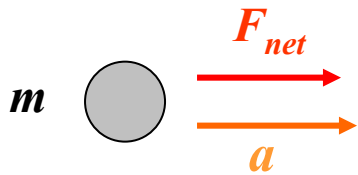
$$\frac{m_X}{m_o} = \frac{a_o}{a_X} \rightarrow m_X = m_o \frac{a_o}{a_X}$$

Thus by measuring a_X we are able to determine the mass m_X of any object.

(5-4)

Newton's Second Law

The results of the discussions on the relations between the net force F_{net} applied on an object of mass m and the resulting acceleration a can be summarized in the following statement known as: "**Newton's second law**"

F_{net}

m

a

The net force on a body is equal to the product of the body's mass and its acceleration

In equation form Newton's second law can be written as:
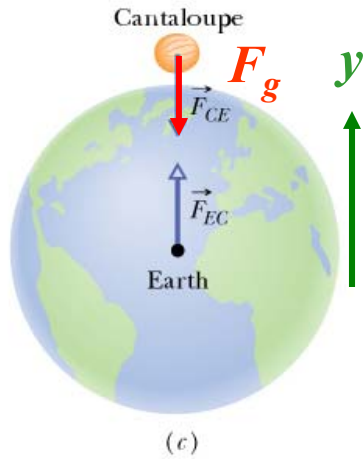
$$\vec{F}_{net} = m\vec{a}$$

The above equation is a compact way of summarizing three separate equations, one for each coordinate axis:

$$F_{net,x} = ma_x$$

$$F_{net,y} = ma_y$$

$$F_{net,z} = ma_z$$

In this section we describe some characteristics of forces we will commonly encounter in mechanics problems

Cantaloupe

F_g y

F_{CE}

F_{EC}

Earth

(c)

The Gravitational Force: It is the force that the earth exerts on any object (in the picture a cantaloupe) It is directed towards the center of the earth. Its magnitude is given by Newton's second law.

$$\vec{F}_g = m\vec{a} = -mg\hat{j} \qquad \left|\vec{F}_g\right| = mg$$

g W y

mg
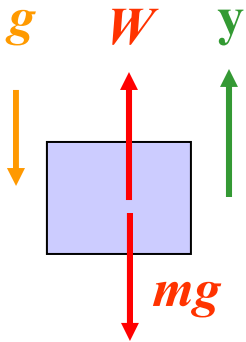
Weight: The weight of a body is defined as the magnitude of the force required to prevent the body from falling freely.

$$F_{net,y} = ma_y = W - mg = 0 \rightarrow W = mg$$

Note: The weight of an object is not its mass. If the object is moved to a location where the acceleration of gravity is different (e.g. the moon where $g_m = 1.7$ m/s^2) , the mass does not change but the weight does.

(5-6)

Contact Forces: As the name implies these forces act between two objects that are in contact. The contact forces have two components. One that is acting along the normal to the contact surface (**normal force**) and a second component that is acting parallel to the contact surface (**frictional force**)
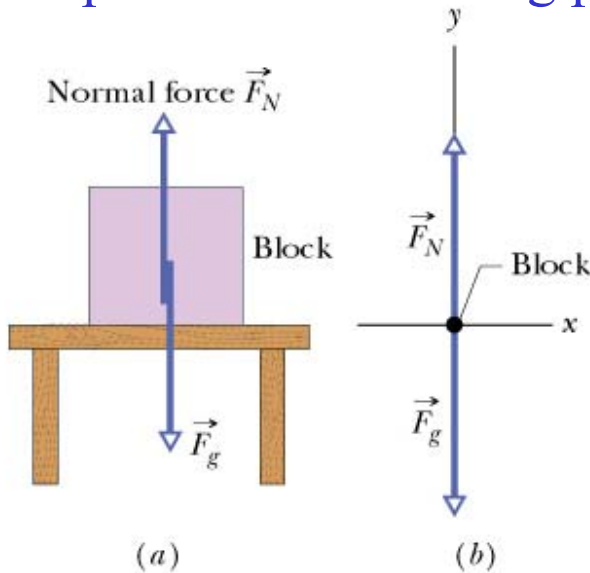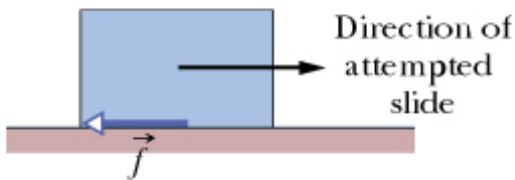
Normal Force: When a body presses against a surface, the surface deforms and pushes on the body with a normal force perpendicular to the contact surface. An example is shown in the picture to the left. A block of mass m rests on a table.

$$F_{net,y} = ma_y = F_N - mg = 0 \rightarrow F_N = mg$$

Note: In this case $F_N = mg$. This is not always the case.

(5-7)

Friction: If we slide or attempt to slide an object over a surface, the motion is resisted by a bonding between the object and the surface. This force is known as "friction". More on friction in chapter 6

Tension: This is the force exerted by a rope or a cable attached to an object
Tension has the following characteristics:
1. It is always directed along the rope
2. It is always pulling the object
3. It has the same value along the rope.(for example between points A and B)
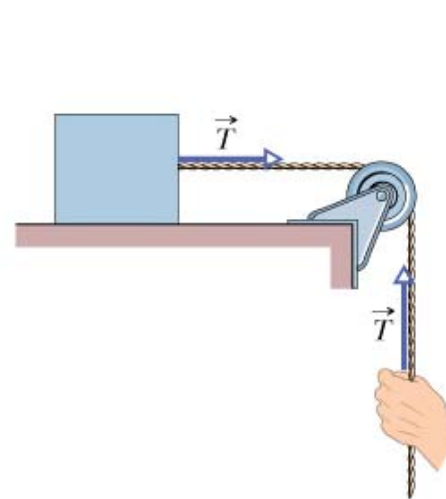
The following assumptions are made:
a. The rope has negligible mass compared to the mass of the object it pulls
b. The rope does not stretch
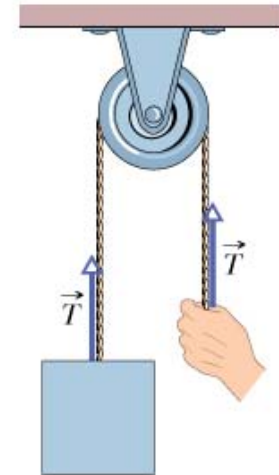If a pulley is used as in fig.(b) and fig.(c), we assume that the pulley is massless and frictionless.
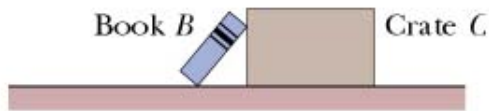
(5-8)

A   B
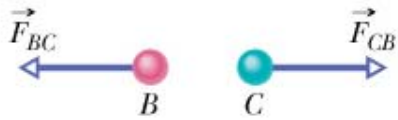
$\vec{T}$ $\vec{T}$ $\vec{T}$ $\vec{T}$ $\vec{T}$ $\vec{T}$
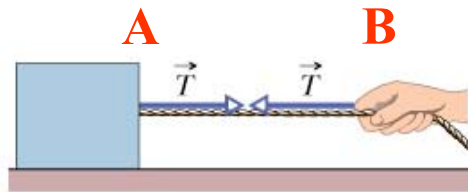
(a)   (b)   (c)

Newton's Third Law:

Book B   Crate C

(a)

$\vec{F}_{BC}$ $\vec{F}_{CB}$

B   C

(b)

When two bodies interact by exerting forces on each other, the forces are equal in magnitude and opposite in direction

For example consider a book leaning against a bookcase. We label $\vec{F}_{BC}$ the force exerted on the book by the case. Using the same convention we label $\vec{F}_{CB}$ the force exerted on the case by the book. Newton's third law can be written as:

$\vec{F}_{BC} = -\vec{F}_{CB}$ The book together with the bookcase are known as a "third-law force pair"

Cantaloupe

$\vec{F}_{CE}$

$\vec{F}_{EC}$

Earth

(c)

A second example is shown in the picture to the left.

The third-law pair consists of the earth and a cantaloupe.

Using the same convention as above we can express

Newton's thir law as: $\vec{F}_{CE} = -\vec{F}_{EC}$

(5-9)

Inertial Reference Frames:

We define a reference frame as "**inertial**" if Newton's three laws of motion hold. In contrast, reference frames in which Newton's law are not obeyed are labeled "**non-inertial**".

Newton believed that such at least one inertial reference frame R exists. Any other inertial frame R' that moves with **constant velocity** with respect to R is also an inertial reference frame. In contrast, a reference frame R'' which **accelerates** with respect to R is a non-inertial reference frame.

(5-10)

The earth rotates about its axis once every 24 hours and thus it is accelerating with respect to an inertial reference frame. Thus we are making an approximation when we consider the earth to be an inertial reference frame. This approximation is excellent for most small scale phenomena. Nevertheless for large scale phenomena such as global wind systems, this is not the case and corrections to Newton's laws must be used.

Applying Newton's Laws / Free body Diagrams

Part of the procedure of solving a mechanics problem using Newton's laws is drawing a free body diagram. This means that among the many parts of a given problem we choose one which we call the "**system**". Then we choose axes and enter **all** the forces that are acting on the system and omitting those acting on objects that were not included in the system.

An example is given in the figure below. This is a problem that involves two blocks labeled "A" and "B" on which an external force $\vec{F}_{app}$ is exerted.
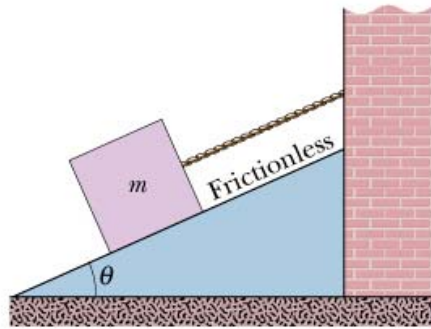
We have the following "system" choices:

a. System = block A + block B. The only horizontal force is $\vec{F}_{app}$

b. System = block A. There are now two horizontal forces: $\vec{F}_{app}$ and $\vec{F}_{AB}$

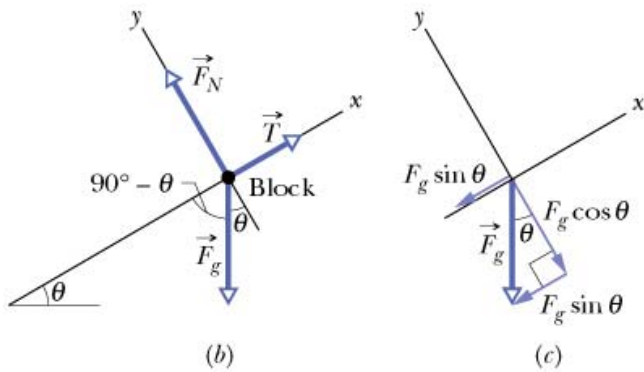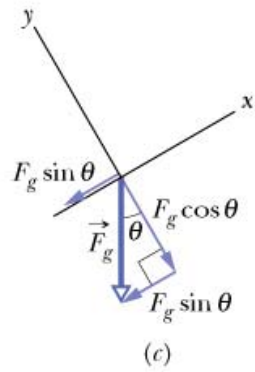c. System = block B. The only horizontal force is $\vec{F}_{BA}$

(a)   (b)   (c)

(5-11)

(a)

(b)

(c)
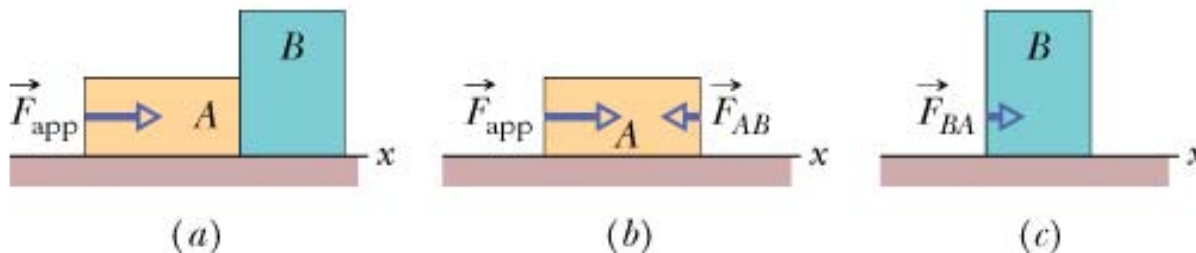
Recipe for the application of Newton's law's of motion

1. Choose the system to be studied
2. Make a simple sketch of the system
3. Choose a convenient coordinate system
4. Identify all the forces that act on the system. Label them on the diagram
5. Apply Newton's laws of motion to the system   (5-12)